Longitudinal diagnostics of electron bunches can be done by measurement of coherent radiation and subsequent extraction of the form factor. By measuring short wavelengths, fine structures in the bunch can be resolved. The form factor depends on the three-dimensional charge density distribution, and the finite transverse extent of the electron bunch can reduce the form factor at short wavelengths. An experimental study using a two stage single shot spectrometer [1] has been carried out at the FLASH free-electron laser at DESY [2, 3]. The coherent transition radiation spectra for two beam optics settings were recorded and compared. In one setting the transverse beam size at the transition radiation target screen has been blown up by a factor of about 3.5 compared to the second setting.

COHERENT RADIATION DIAGNOSTICS

A bunch of \( N \) electrons radiates a spectrum according to

\[
\frac{d^2U}{d\lambda d\Omega} = \left( \frac{d^2U}{d\lambda d\Omega} \right)_1 \left( N + N(N-1) \left| F_{3d}(\vec{k}) \right|^2 \right),
\]

where the three-dimensional bunch form factor is defined by

\[
F_{3d}(\vec{k}) = \int S_{3d}(\vec{r}) e^{-i\vec{k} \cdot \vec{r}} d\vec{r}.
\]

\( \vec{k} \) is the wave vector towards the observation point. The normalized three-dimensional charge distribution is given by the ensemble average of the electron distribution,

\[
S_{3d}(\vec{r}) = \frac{1}{N} \left( \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i) \right).
\]

Most often only the longitudinal form factor in forward \( (z) \) direction is considered. Taking into account only the \( z \) component of the wave vector, it is

\[
F(\lambda) = \int_{-\infty}^{\infty} S(z) \exp \left( \frac{-2\pi i z}{\lambda} \right) dz,
\]

with the longitudinal charge distribution

\[
S(z) = \int S_{3d}(\vec{r}) dx dy.
\]

However, a finite transverse extent of the bunch can affect the coherent amplification at short wavelengths and reduce

\[\text{Figure 1: Illustration of the effect of a finite transverse size on the form factor magnitude}\]

the magnitude of the form factor. The effect can be estimated by considering the case of a longitudinal and transverse Gaussian charge distribution with rotational symmetry about the \( z \) axis,

\[
S_{3d}(x, y, z) = \frac{1}{2\pi\sigma_x^2} e^{-\frac{x^2}{2\sigma_x^2}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{z^2}{2\sigma_z^2}}.
\]

The form factor (2) can be evaluated explicitly in this case:

\[
F_{3d}(k_x, k_y, k_z) = e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{z^2}{2\sigma_z^2}}.
\]

Here, \( k_x^2 + k_y^2 = (2\pi \sin \alpha / \lambda)^2 \) and \( k_z = 2\pi \cos \alpha / \lambda \) for an observation angle \( \alpha \) with respect to the \( z \) axis. The form factor is reduced to \( 1/e \) of its maximum value (obtained for an infinitely thin line bunch) for a transverse size \( \sigma_t = \lambda/(\sqrt{2\pi}\sin \alpha) \). These relations are illustrated in Fig. 1.

The equation for \( F_{3d} \) shows that the transverse contribution to the form factor is determined by \( \sigma_t \sin \alpha \), the longitudinal by \( \sigma_z \cos \alpha \). For small angles, typical for radiation from highly relativistic electrons, transverse effects are therefore strongly suppressed.

MEASUREMENT SET-UP

The measurements were performed with coherent transition radiation emitted by a radiator located in a diagnostic section of FLASH after the accelerating modules. The radiation was transported to an experimental hut outside of...
the radiation controlled area over a distance of 18 m by a THz beam-line [5]. The interface to the ultra-high vacuum of the accelerator is a diamond window.

A two stage single-shot grating spectrometer [1] has been used in the measurement. Diagnostics of the ultra-short electron bunches by means of spectroscopy of coherent radiation, down to near-infrared, had been already demonstrated using this device.

The entire spectroscopy of coherent radiation is performed in vacuum to avoid water vapour absorption.

MATLAB TOOLBOX FOR ELECTRON-BEAM OPTICS

Tuning of the electron beam optics was started from the optics established for the SASE operation (original optics). To blow up the beam size at the screen (approximately by a factor of 5 in both planes) the five upstream quadrupoles were tuned according to the solutions calculated using the MatLab toolbox available at FLASH for the online calculation of the linear beam optics.

MEASUREMENT

The FLASH accelerator was tuned to provide high intensities at micrometer wavelengths with standard user operation optics. The measured average spectrum is shown in Fig. 6 in blue color. In the next step, by adjusting the settings of several quadrupoles according to the beam optics calculation, the transverse size of the electron beam has been blown up by a factor of $\approx 3.5$ at the OTR screen $18$ACC7. The averaged measured spectrum under this condition is shown in red. The ratio of these two measured spectra is shown in green and the theoretical calculation from (5) in dashed-black. The theoretical calculation assumed a flat single-electron spectrum. The influence of the reduced charge transmission of the blown-up beam is taken into account in Fig. 6. Each measurement point is the average of 400 shots.

The difference between the measured and theoretical curve at wavelengths below about 12 $\mu$m is presumably due to changes in the longitudinal charge distribution resulting from path length changes between the two optics settings. Also the assumption of three-dimensional Gaussian profiles in the calculation is not fulfilled in reality.

$^1$To achieve a better match with the desired factor of 5, a more detailed initial study of the original optics would have been necessary.
CONCLUSION

This experiment shows a qualitative agreement with the theory which can be considered as the basis for further investigations. The effect of the transverse size of the electron bunch, especially when short structures are the subject of coherent radiation diagnostics, is demonstrated.

REFERENCES