SELF-CONSISTENT TRANSVERSE DYNAMICS AND INTERBUNCH ENERGY EXCHANGE IN DIELECTRIC LOADED WAKEFIELD ACCELERATING STRUCTURES *

I. L. Sheynman #, Saint-Petersburg Electrotechnical University (LETI), Saint-Petersburg, Russia
A. D. Kanareykin, ANL, Argonne, IL 60439, U.S.A.

Abstract

The self-consistent transverse dynamics of high current relativistic electron beams used for generating wake fields in dielectric loaded structures is investigated. The primary application of this work is to multi-bunch wakefield acceleration. The maximum distance the high current beam can travel through the structure in the absence of focusing has been simulated. The BBU (beam break-up) effects and interbunch energy transfer within accelerated electron bunch have been studied as well. We consider ramped charge distribution in the sequence of high current drive bunch. This ramped drive charge distribution with different summary charges and bunch lengths are compared in terms of the beam focusing system requirements and the efficiency of the energy transfer to the accelerated electron bunch.

MULTIBUNCH WAKEFIELD ACCELERATION SCHEME

The technology of dielectric wakefield electron accelerators employing the Vavilov–Cherenkov effect is one of the most promising directions in the development of high-field-gradient structures for modern linear accelerators, and it has been extensively studied in recent years [1–4]. The main element of such an accelerating structure is a cylindrical metal waveguide filled with a dielectric and having an axial vacuum channel. A short (1–4 mm) driving electron bunch possessing a large charge (20–100 nC) travelling along the channel generates a TM_{01} wakefield mode of the Cherenkov radiation with a longitudinal electric field component. The subsequent high-energy witness (driven) electron bunch with a small charge, following the former bunch at a certain delay selected to fit the accelerating phase of the wakefield, is to be accelerated by this field up to 0.1–1 GeV energy.

The factors of key importance in wakefield acceleration is an increase in the energy transformation ratio R and the accelerating wakefield E^{+}, which are the parameters characterizing the efficiency of the acceleration process [1, 3]. The R value is defined as the ratio of the maximum electron energy increment in the witness bunch to the maximum electron energy loss in the driving bunch.

The traditional wakefield accelerator implements a collinear scheme, in which the driven and driving electron bunches travel along the same axis. However, collinear systems are characterized by limited total energy increment for the driven bunch. In accordance with the wakefield theorem [6], electrons in the driven bunch cannot increase their energy by an amount exceeding doubled electron energy in the driving bunch. Accordingly, wakefield accelerators implementing the scheme with electron bunches moving along the same axis (with a symmetric driving bunch) have $R \leq 2$.

To increase the energy transformation ratio and the accelerating wakefield multibunch schemes are used. In order to provide for $R > 2$, a train of Gaussian bunches with a period of d having a triangular envelope (ramped bunch train, RBT) is used. The charge in this train varies from a minimum for the first bunch to a maximum for the last bunch, the four leading bunches in the train are subjected to the action of a relatively low retarding field of the same amplitude. A high efficiency of the energy transfer from the train of driving bunches to the field in the RBT mode is achieved at the expense of increasing distance travelled by these bunches.

TRANSVERSE DYNAMICS

Selection and development of the acceleration scheme depend to a significant extent on the presence of transverse deflecting fields. An analysis of the transverse dynamics for a train of high-current bunches shows that a radial deviation of the beam poses limitations on the distance l travelled by bunches in the waveguide [4]. An excess deviation caused by the deflecting forces results in that the beam interacts with the waveguide wall and further acceleration becomes impossible. Therefore, the transverse instabilities restrict the possibility of gaining energy from the train of driving bunches.

The use of RBT acceleration scheme with the condition for ideal suppression of the lateral beam instabilities by the focusing system makes possible to reach the practically complete extraction of kinetic energy of the driver bunches and transfer it for accelerated bunch. In practice focusing only partially suppresses the lateral instabilities, leading to a relative increase of the flying range in comparison with the case without the use of the external focusing system. As a result the electrons of bunches fall to the waveguide wall and do not manage to transmit their energy to wake field.

The energy transmitted to the accelerated bunch $\Delta W^{+} = \int eE^{+} dl$ decreases with the growth of the charge losses of the accelerating beam. The spatial distribution of charge renders influence not only on the accelerating gradient E_z^{+} and transformation ratio R, but it determines

* This work is supported by Russian fond of Basic Research, grant № 06-02-16442
# isheinman@yandex.ru

also the radial fields, which act on the bunches. As a result, flying range also proves to be depending on the selection of charge distribution in the chain of the driver bunches.

It should be noted that in RBT scheme the charge magnitudes of the accelerating bunches and distance between them are determined by the structure of scheme [3]. In the single-mode regime of the excitation of wakefield waveguide by four bunches (if $\sigma > 0.2\lambda$) the relationship between charges comprise 1:3: 5: 7.

In the sequence of bunches each subsequent driver bunch is placed into the maximum of the field, created by the previous bunches, moreover summary field after all bunches increases.

BEAM DYNAMICS EQUATIONS

The fields calculation we will carry out under the assumption Gaussian charge distribution in bunches both on longitudinal $f(z)$ and on radial $f(r)$ coordinates. Since into the deflection field with the small deviations of beam from the axis the greatest contribution introduces the first azimuthal mode, the force, which acts on the charges in the radial direction, depends on r linearly ($I_1(kr) \approx kr$ with small kr), integration of the elementary sources of transverse force for the radial coordinate gives the average of Gaussian distribution.

Thus, for radial dynamics calculating it is possible to consider that the charge is concentrated in the center line of the transverse distribution of bunch [4]. Subsequently we will examine the filamentary electronic beam with the longitudinal charge distribution $f(\zeta)$, which moves along the axis of waveguide with the displacement $r(\zeta, t)$. Let us take initial displacement of beam $r(\zeta, t)\big|_{t=0} = r_0$.

The equations of longitudinal and radial dynamics with the relativistic speed of beam in the vacuum are [3]:

$$F_z = eE_z = m_e \frac{d\left(v_z\gamma\right)}{dt},\ F_r = e\left(E_r - v_z B_\theta\right) = m_e \frac{d\left(v_r\gamma\right)}{dt},$$

Longitudinal and radial forces are obtained by integrating the function, which describes the field of emission at point z, r from the point charge, which is located at point with the coordinates z_0, r_0, of that convoluted with the function of the longitudinal bunch charge distribution.

Integrating the equations of longitudinal and radial dynamics for the time and solving the obtained system relative to longitudinal and radial velocity, we will obtain:

$$\beta_z(\zeta) = \frac{\xi(\zeta)}{\sqrt{1+\xi(\zeta)^2+\eta(\zeta)^2}};\ \beta_r(\zeta) = \frac{\eta(\zeta)}{\sqrt{1+\xi(\zeta)^2+\eta(\zeta)^2}},$$

where $\beta_z = v_z/c$; $\beta_r = v_r/c$; $\zeta = z - v_z t$;

$\xi(\zeta) = a_z(\zeta)t + \beta_{z0}\gamma_0$; $\eta(\zeta) = a_r(\zeta)t + \beta_{r0}\gamma_0$;

$a_z(\zeta) = eE_z(\zeta)/\left(m_e c\right)$; $a_r(\zeta) = F_r(\zeta)/\left(m_e c\right)$.

After producing repeated integration, we will obtain the dependence of the radial displacement of the centre line of beam on the axis of waveguide from the position of particle in the bunch $r(\zeta, t)$.

BEAM DYNAMICS SOLVING

A program for solving the equations of the self-consistent dynamics of the beam, which consist of the four accelerating bunches and one accelerated bunch, is developed.

As the basis of the program is assumed the modified algorithm [4]. At each time moment is known the dependence $r_0(\zeta, t)$ of the radial displacement of the bunch particles which it is used for calculating the fields. For determining the radial displacement of the center line of bunch in next time moment $r(\zeta, t+\Delta t)$ its partition into the chain of macroparticles is produced.

On the basis of the given above analytical solution of motion equations (in contrast to [4], where the differential equations are transferred into the difference form with the accuracy of the solution of 1 orders) the coordinates of macroparticles along the center line of bunch are calculated at the subsequent moment of time, and then this discrete sequence is interpolated for obtaining the newly functional dependence $r(\zeta, t)$.

The results of the work of program are: field structure after the bunch, the form of the longitudinal-radial distribution of the system of bunches at the arbitrary moment of time or before the contact by the beam of waveguide wall, dependence of that acquired and lost by the bunches of energy ΔW for the end point of the time of calculation before the contact by the beam of the wall of waveguide. The flying range L of the system of bunches (with number to 5) before the contact of wall, which defines the moment of the interruption of calculation, can be found both taking into account and without taking into account the focusing force.

In the idealized situation of “absolute focusing”, a maximally possible flying range $L_{\max}$ is found from the condition for the disturbance of field phase synchronization in bunches chain with starting energy W_d. The deceleration of the driver bunches by the field E^- and acceleration of the witness, creates between them into the phase difference, equal to half of the wavelength of the base accelerating mode of waveguide [3].

$$L_{\max} = \frac{\lambda W_d^2}{(m_0 c^2)^2 + \lambda W_d e E^-}.$$

The flying range $L_{\max}$ can be achieved with the condition for complete suppression by the focusing system of the lateral instabilities of electron beam.

As a rule, in the high current multibunch acceleration schemes the introduction of external focusing does not make it possible to attain the complete suppression of the lateral instabilities of beam and leads only to certain increase in the flying range prior to the contact by the bunches of the wall of waveguide. The quality of focusing can be described by the parameter α, defined as the ratio of the flying range of beam with focusing to the flying range of beam without it. The parameter of $\alpha_{\max}$ given below in a table corresponds to the maximum flying distance $L_{\max}$.

SIMULATION RESULTS ANALYSIS

Calculations were performed for the waveguide with the parameters: outer waveguide radius $R_w = 0.6342$ cm, inner radius $R_c = 0.5$ cm, dielectric constant $\varepsilon = 16$, base frequency $f = 13.625$ GHz, which correspond to the parameters of the test accelerator of the Argon National Laboratory of the USA [2].

Radial charge distribution at the moment of contact by the beam of the wall of waveguide is shown in Fig. 1. It is evident that the radial displacement increases with the number of the bunch.

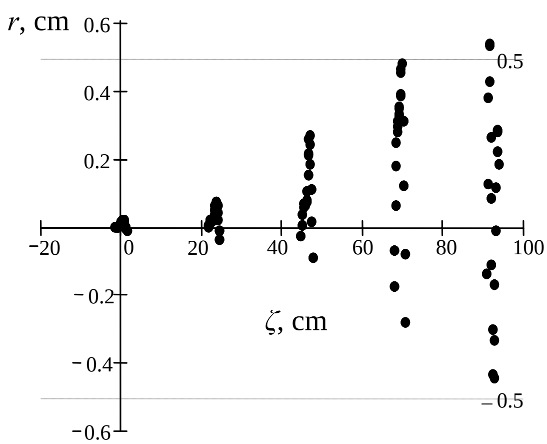

Figure 1: Radial charge distribution

In the table for the beam of four driving bunches and one witness bunch with the summary charge Q_Σ, the mean-square length of σ_z and starting energy W_d calculated values of wake field E_z, transformation ratio R and energy acquired by the accelerated bunch W_a are presented. The flying ranges of the system of the bunches before the contact of the wall of waveguide L, maximally possible flying ranges L_{max} and corresponding to them parameters of the quality of the focusing of α_{max} also are given.

The initial deviation of beam from the axis of waveguide accepted by identical and composes $r_0 = 0.01$cm. The charge distribution and distances between bunches are selected from the condition of the maximization of the transformation ratio.

When $\sigma_z = 0.4$ cm the single-mode regime of the excitation of waveguide is ensured, when $\sigma_z = 0.15$ cm the waveguide is excited in the multimode regime. For planning driver bunch energy $W_d = 100$ MeV the focusing system quality α_{max} should be in a range 30…100, but calculated and achieved values are less then 10. It is open an area to new investigations.

In the accelerating structure on the basis of dielectric a maximally possible accelerating field is, as a rule, limited by dielectric strength of the material of filling. In the last table column are given the results of calculations for the short bunches of $\sigma_z = 1.5$ mm, which ensure the maximum accelerating field in the waveguide, not creating electrical breakdown in the dielectric (accepted for the calculations E_z=100 MV/m). A similar gradient was recently demonstrated in the experiment on the wakefield accelerator AWA.

Parameter	**One mode, AWA now**	**Multi mode, AWA now**	**One mode, plan energy**	**Multi mode, plan energy**	**Multi mode, max field**
Q_Σ, nC	100	100	100	100	918
σ_z, cm	0.4	0.15	0.4	0.15	0.15
W_d, MeV	15	15	100	100	100
E_z, MV/m	3.71	10.93	3.72	10.75	100
R	7.63	5.30	7.62	5.13	5.32
W_a, MeV	2.53	6.69	6.3	16.7	50.9
L, m	0.64	0.63	1.6	1.6	0.52
L_{max}, m	11.7	5.2	165	45.1	5.29
α_{max}	18.4	8.4	103	28.1	10.2
$W_{a\ max}$, MeV	46.5	56	648	469	519

For the manifestation of the advantages of RBT acceleration scheme in the energy transfer from the accelerating bunches to the accelerated bunch it is necessary to ensure a sufficient flying distance, at which occurs interaction of beam with the accelerating structure. For the majority of the cases this requires the introduction of the external focusing of electron beam (as a rule, quadrupole) [4]. It should be noted that for any type of wakefield accelerator the length of structure must be close to a maximally possible length of interaction of beam with the structure, which is necessary for the effective energy takeoff from the drive bunches and the transfer of this energy to the accelerated beam. The requirements for the focusing system for drive bunches, formulated in this article, present conditions to the effective length of the flight of bunches in the structure.

REFERENCES

[1] J. G. Power, W. Gai, A. Kanareykin "Transformer Ratio Enhancement Using A Ramped Bunch Train In A Collinear Wakefield Accelerator", AAC'2000, AIP Conference Proceedings 569, NY, p. 605-615, (2001).

[2] M. Conde "Survey of Advanced Dielectric Wakefield Accelerators". Proceeding PAC'2007, Albuquerque, NM, USA, p. 1899-1903 (2007).

[3] I. L. Sheinman and A. D. Kanareykin. "Criteria for the Optimization of Parameters of a Multibunch Scheme of Wakefield Acceleration", Technical Phys. Lett., 31(4), p. 325–328 (2005).

[4] King-Yuen Ng. Single-Bunch Beam Breakup in a Dielectric-Lined Waveguide / Proceedings XVth International Conference on High Energy Accelerators (FERMILAB-Conf-92/212), Hamburg, Germany, July 20-24, 1992.