

FFAGS AS MUON ACCELERATORS FOR A NEUTRINO FACTORY

S. Machida, ASTeC CCLRC RAL, U.K.

Abstract

The Fixed Field Alternating Gradient (FFAG) accelerator is a solution for rapid acceleration of muons because of its large aperture and no need of magnet ramping. Its particle dynamics is, however, peculiar due to high energy gain per turn and large transverse amplitude, which has not been seen in other types of circular accelerators. One variation of FFAG, called non-scaling FFAG, employs quite new scheme, namely, out of bucket acceleration. We studied emittance distortion, coupled motions among 3-D planes, effects of resonance lines based on a newly developed tracking code. In this paper, we will emphasize new regime of particle dynamics as well as a modeling technique of FFAG.

CODE DEVELOPMENT

Requirements

Tracking codes for a synchrotron usually assume reference orbit and momentum. Magnets are aligned such that a particle with reference momentum goes through the centre. Off-momentum particle motion is approximated with an assumption of small deviation.

In Fixed Field Alternating Gradient (FFAG) accelerator, on the other hand, “reference” does not make any meaning because magnetic field is constant and orbit moves as momentum increases. Position of magnets is independent of any particular orbit. Although it is still possible to assume the central momentum as a reference one, the assumption of small deviation easily fails because the momentum deviation is such as -33% to 33% or more.

Tracking code for FFAG should first separates position of magnet and orbit. Secondly it must model a particle orbit with wide momentum range without any approximation. We also know that end fields effects are not negligible because of wide gap of FFAG magnets. Tracking code should have proper model of them.

Modeling

We took a naïve approach to model FFAG. First, we translate all elements to thin lens representation. Since end field is not negligible and strength in the body region sometimes strongly depends on its longitudinal position, we take an area well extended from a magnet body and the whole region is represented by a bunch of thin lenses. The strength of each thin lens is determined by analytic model such as Enge function or 3-D field calculation by TOSCA. Eventually it should be determined by field measurement data.

Position and strength of magnets are first determined by simplified model with a synchrotron code such as MAD [1] and SAD [2]. For example, we use a combined

function type magnet to simulate FFAG magnet. In scaling FFAG, strength has to be adjusted looking at tracking results to make a preferable footprint on tune diagram. In non-scaling FFAG, position has to be adjusted to have a right behavior of time of flight. Focusing strength is less critical because tune moves in a large range.

Integration

Tracking of particle is literally done by integrating an equation of motion with the independent variable of time. In each time step of Δt , necessary time of t_l to reach the next thin lens from the present position is first calculated. If t_l is more than Δt , a particle falls in between depending on the ratio of $\Delta t/t_l$. If t_l is less than Δt , the particle receives transverse kick from the next thin lens element.

$$P_{y,new} = P_{y,old} + \frac{1}{1+\delta} \left(b_x \frac{P_{z,old}}{P_{x,old}} - b_z \right) \Delta x, \quad (1)$$

$$P_{z,new} = P_{z,old} + \frac{1}{1+\delta} \left(b_y - b_x \frac{P_{y,old}}{P_{x,old}} \right) \Delta x, \quad (2)$$

$$P_{x,new} = \sqrt{P_{old}^2 - P_{y,new}^2 - P_{z,new}^2}, \quad (3)$$

where y is horizontal, z is vertical, and x is longitudinal. b is strength of magnet, δ is deviation of momentum from the central value and Δx is distance between thin lenses. Note that longitudinal momentum is updated after two transverse ones are calculated to keep the total momentum constant.

At RF cavity, only longitudinal momentum is usually updated. However, when RF cavity is installed in such a way that a particle does not go through parallel to the electric field lines, for example in spiral FFAG, transverse momentum may be increased as well.

Benchmark

The tracking code is benchmarked with Zgoubi. We compare time of flight for zero as well as finite transverse amplitude. They agree each other quite well [3].

NON-SCALING FFAG

Acceleration method and phase slip problem

In the momentum region where FFAG is employed to accelerate muons, particle motion is already relativistic and time of flight is determined solely by path length. Non-scaling FFAG optics is designed such that path length variation from injection to extraction momentum is minimized [4]. In such a way, transverse orbit excursion is also minimized resulting in small magnet aperture. Furthermore, when the variation of time of flight is minimized, phase slip relative to the RF crest becomes

small. Muon acceleration finishes within 10 to 20 turns with a huge energy gain. Therefore, with a proper choice of RF frequency, a particle can be accelerated almost always at the crest of RF wave even if there is still small phase slip. Acceleration out of bucket is proposed as a way to accelerate muons in non-scaling FFAG [4].

The scheme works fine for the small transverse amplitude particle. However, large amplitude one such as 30π mm (normalized), which is in fact the design emittance of neutrino factory, path length dilation due to transverse oscillations becomes significant. In the other words, in the longitudinal phase space, large amplitude particles lag behind small ones, which develops emittance tail. In some case, such a particle cannot stay in the region of positive voltage and falls into deceleration phase in a middle of acceleration.

Figure 1 shows longitudinal phase space for zero and finite transverse emittance beam. We use lattice and beam parameters of 10 to 20 GeV muon ring. Dilution of longitudinal emittance is obvious and that is not acceptable.

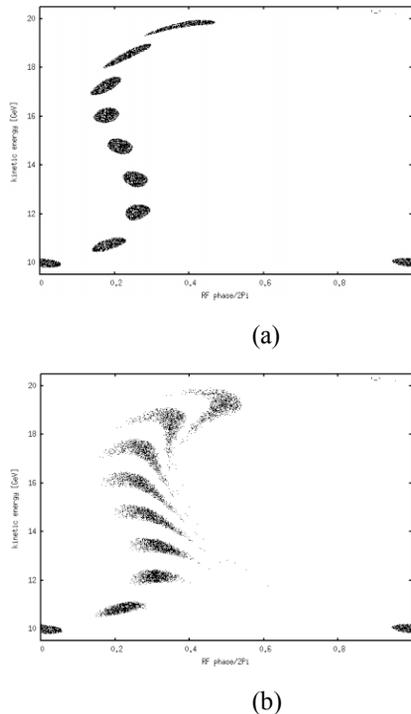


Figure 1: Footprint of longitudinal phase space from 10 to 20 GeV muon beam with (a) zero and (b) finite transverse amplitude.

In principle, the same effects of path length dilation exist in other type of accelerators. In non-scaling FFAG for muons, however, they are emphasized because of huge emittance, 30π mm, and acceleration outside of a bucket. In particular, the latter ceases synchrotron oscillations so that phase spread increases monotonically once a particle lags behind the other.

Possible cures

There are several ways to cure the problem. Obviously, increasing the RF gradient and accelerating muons with less number of turns mitigate the total phase slip and therefore longitudinal emittance growth. That is straightforward but increase of RF power simply means more cost. Recalling that FFAG is cost effective because of its multiple use of RF, the attempt to increase RF voltage is less favored. Decreasing RF frequency is another way to reduce the total phase slip. Time difference between two particles is the same, but RF phase difference is relatively reduced. The problem is RF source and gradient in lower frequency. With lower frequency, available RF gradient tends to be lowered because of spark limit. As a rule of thumb, it is inversely proportional to the square root of frequency. Although it still gains, lower gradient means more time for acceleration to the final energy. Overall muon yields becomes a concern.

We found the most effective way to solve the problem is an introduction of higher harmonic RF. By adding second or third harmonic wave on the top of fundamental RF crest with opposite sign, we can flatten the RF waveform. Therefore energy gain become less dependent on RF phase. Although phase slip still exists, that does not make huge spread of momentum direction as before.

Further tracking study for optimization is foreseen. First of all, introduction of higher harmonic RF is meant to be additional cost. Second harmonic has to have 25% of fundamental voltage to make the crest flat. That is not negligible cost increase. Probably, some combination of other means with higher harmonic RF gives the optimized design. Secondly, at least now, we are considering two FFAG in a cascade. Fixing the problem in the first FFAG is not good enough for the second FFAG. In fact, the correlation of momentum and phase introduced in the first ring enhances the same problem in the second ring. Figure 2 shows longitudinal phase space of two cascade FFAG with second harmonic RF and 10% more peak RF voltage.

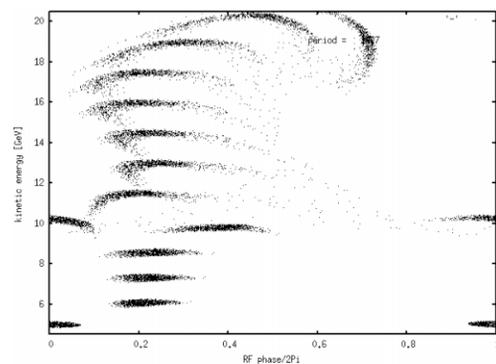


Figure 2: Footprint of longitudinal phase space from 5 to 10 GeV first ring and 10 to 20 GeV second ring.

Resonance crossing

Non-scaling FFAG has to cross many resonance lines during acceleration because lattice consists of only dipole and quadrupole. Ratio of phase advance at low momentum and at high momentum is, for example, more than two when the momentum range is twice. It is possible to avoid structure resonances of integer and half integer, but hard to do higher order above half integer. Figure 3 shows tune per cell of 10 to 20 GeV muon ring. Nevertheless, a very fast crossing rate such as one unit of total tune per turn may make the resonances invisible. Furthermore, nonlinearity comes only from kinematic effects and end fields so that source of those resonances is supposed to be small.

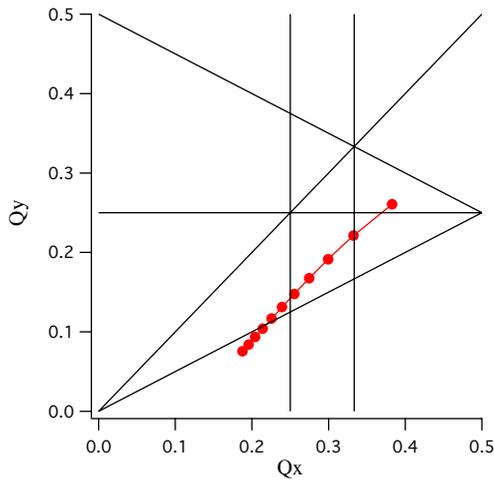


Figure 3: Tune diagram of 10 to 20 GeV muon ring. Filled circles indicate tune at 10, 11, ..., 20 GeV.

Particle tracking has been done first to look at the effects of higher order structure resonances. There are $3\nu_x=1$, $\nu_x-2\nu_y=0$, and $4\nu_x=1$ resonances among other nonlinear ones. We took artificial model of longitudinal dynamics to avoid phase slip problem mentioned above. Namely, energy gain does not depend on RF phase and all the particles reach the final energy in 17 turns. In that way, resonance crossing of a 30π mm beam can be simulated.

As shown in Fig. 4, emittance exchange occurs when a beam cross $\nu_x-2\nu_y=0$ resonance. That emittance exchange is, however, not very serious. If there is 8% larger aperture compared with emittance, 2% of the particles hit the aperture. If it is 15% larger, 0.2% becomes beam loss, which is negligible. Other resonances do not make any problem. The source of the resonance is likely coming from kinematic terms due to large emittance because the results do not change whether we include end fields with Enge model or not.

Once the periodicity of the lattice breaks due to alignment errors or field imperfections, many non-structure resonances appear, which includes integer and half integer resonances. Another particle tracking study

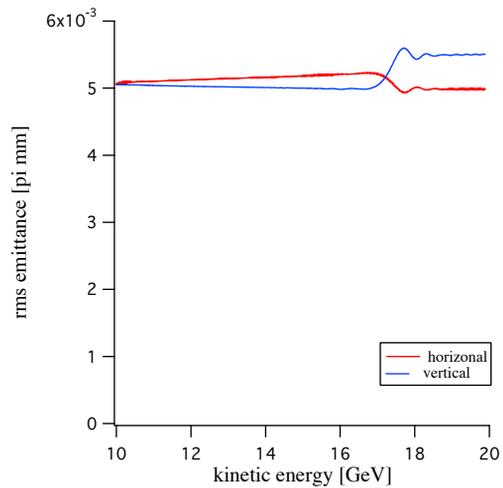


Figure 4: RMS emittance evolution of 10 to 20 GeV muon ring. Waterbag distribution of 30π mm is launched.

including alignment errors was carried out to see how much beam loss occurs. The aperture limit is set at 45π mm and a 30π mm beam is accelerated. As alignment error, rms values of 5, 10, 20, 50, 100, 200, 500 μ m are assumed. Distribution is Gaussian and the maximum value is twice of rms. Figure 5 shows the survival for different alignment errors. To keep the beam loss less than 10%, alignment has to be done with less than 20 μ m.

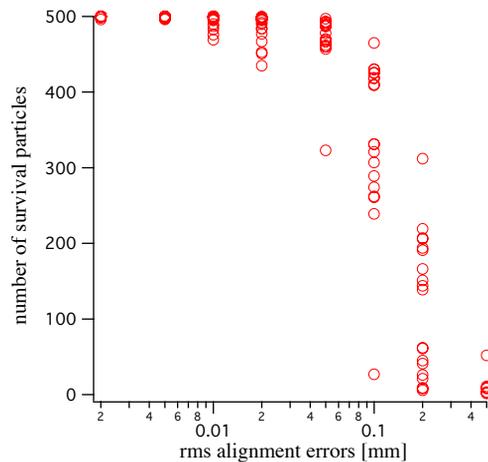


Figure 5: Effects of misalignment. 20 seeds are taken for each error. Loss is defined at 45π mm for 30π mm beam.

REFERENCES

- [1] MAD home page, <http://mad.web.cern.ch/mad/>
- [2] SAD home page, <http://acc-physics.kek.jp/SAD/>
- [3] J. Fourier and S. Machida, "Comparison of Zgoubi and S-Code regarding the FFAG Muon acceleration", to be published.
- [4] US_Study_IIa, <http://www.cap.bnl.gov/mumu/study2a/>