CONTROL OF DYNAMIC APERTURE WITH INSERTION DEVICES*

T. Shaftan#, J. Bengtsson and S. Kramer,
BNL-NSLS, Upton, NY 11973, U.S.A.

Abstract

It is well known that insertion devices (IDs) perturb the linear optics. In particular, that the effect can be corrected locally by a symmetric arrangement of four quadrupoles on each side of the IDs. We present a method to control an arbitrary set of IDs using the response matrix for the beta-beat and phase-beat with SVD, to maintain the dynamic aperture (DA). We also evaluate the residual impact on the DA from the nonlinear terms. We discuss the impact of ID’s on the NSLS-II dynamic aperture. Results for single ID’s and a set of 15 ID’s with random field strengths are presented.

INTRODUCTION

Hamiltonian of a single planar ID with the fields given by the formulas in Ref. [1] can be represented by the following expression [2, 7]:

\[
\langle H \rangle_u \approx \frac{p_u^2 + p_v^2}{2(1 + \delta)} - \frac{y^2}{4\rho_u^2(1 + \delta)} - \frac{k_y^2 y^4}{12\rho_u^4(1 + \delta)} - \delta + O(p_{u,v}^4) 
\]  

(1)

where H is averaged over a single ID period, \( \lambda_u \) is the bending radius in the peak field, and \( \alpha \) is the average \( \rho_u^2 \) along the ID of length \( L_{ID} \). The second term in Eq.(1) corresponds to the vertical focusing induced by ID. It perturbs the ring lattice functions from their design values, creating \( \beta \)- and phase (\( \mu \))-beating around the ring. The nonlinear driving terms [3] depend on values of the lattice functions at locations of the nonlinear elements; this perturbation destroys the careful cancellation of the driving terms, impacting the DA. The linear effect of an ID on the ring optics can be estimated by the vertical tune shift it creates, and is given by:

\[
\Delta \nu_y \approx \frac{\beta_{\nu_y}}{8 \pi \rho_{\nu_y}^2} L_{ID},
\]  

(2)

where \( \beta_{\nu_y} \) is the average \( \beta \) along the ID of length \( L_{ID} \).

The third and higher terms in Eq.(1) drive resonances like \( 2 \nu_y \) and \( 4 \nu_y \) and generating vertical tune shift with amplitude. From Eq.(1), linear and non-linear terms are strong functions of \( \lambda_u \) and \( \rho_u \), therefore the strong-field short-period ID’s, which are required for high-brightness radiation sources in NSLS-II will greatly impact the DA.

To minimize the effects from ID’s we propose to optimize the \( \beta \)-functions in the ID straight, by minimizing the linear and non-linear tune shifts, i.e. the \( \beta \)- and \( \mu \)-beating of the lattice. We estimated the \( \beta \)- and \( \mu \)-beat tolerances for the NSLS-II ring DA by simulating the tolerance for these perturbations by using quadrupole gradient errors and specifying the tolerance for these terms that starts to impact the DA strongly. These tolerances are listed in Table I [3].

<table>
<thead>
<tr>
<th>ID</th>
<th>( \lambda_u ) (mm)</th>
<th>K</th>
<th>( L_{ID} ) (m)</th>
<th>( \Delta \nu_{\nu_y} \times 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCU</td>
<td>14</td>
<td>2.2</td>
<td>2</td>
<td>6.76</td>
</tr>
<tr>
<td>MGU</td>
<td>19</td>
<td>2.2</td>
<td>3</td>
<td>5.50</td>
</tr>
<tr>
<td>DW</td>
<td>100</td>
<td>13.6</td>
<td>6</td>
<td>15.2</td>
</tr>
</tbody>
</table>

Finally we estimate linear impact of the proposed ID’s for NSLS-II. These consist of Super-Conducting (SCU) and Mini-Gap Undulators (MGU), as well as Damping Wigglers (DW). The parameters for these ID’s together with their linear impact on the NSLS-II lattice [4] are presented in Table II.

Table II: ID parameters proposed for NSLS-II and their impact on the linear optics.

<table>
<thead>
<tr>
<th>ID</th>
<th>( \lambda_u ) (mm)</th>
<th>K</th>
<th>( L_{ID} ) (m)</th>
<th>( \Delta \nu_{\nu_y} \times 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCU</td>
<td>14</td>
<td>2.2</td>
<td>2</td>
<td>6.76</td>
</tr>
<tr>
<td>MGU</td>
<td>19</td>
<td>2.2</td>
<td>3</td>
<td>5.50</td>
</tr>
<tr>
<td>DW</td>
<td>100</td>
<td>13.6</td>
<td>6</td>
<td>15.2</td>
</tr>
</tbody>
</table>

CONTROL OF LINEAR OPTICS WITH ID

The procedure we developed was first to correct the linear optics perturbation of the ID locally, by using a set of four quadrupoles (quadruplet-QD) on either side of the ID straight section. Figure 1 shows the NSLS-II lattice functions with a 3m-MGU in the 5m straight.

![Fig. 1: One period of the NSLS-II lattice with a 3m MGU (green bar) shown, together with the quadruplets.](image)

For optics correction we have developed the following method [5]. As been mentioned in the Introduction, DA suffers from perturbations of beta-functions in the ring sextupoles. Therefore our goal is to change strengths of the quadruplets bounding the ID straight in order to minimize the perturbations. We obtain values of horizontal and vertical beta and phase-beat around the
ring by computing lattice functions in all sextupoles for bare lattice and that with an ID. Next we solve system of $4 \cdot N_{\text{sext}} + 2$ equations (4) using SVD and finding increments of $K_1$ for each correcting quadrupole.

$$
\begin{bmatrix}
(\Delta \beta_x / \beta_x)_{\text{SEXT}} \\
(\Delta \beta_y / \beta_y)_{\text{SEXT}} \\
(\Delta \mu_x)_{\text{SEXT}} \\
(\Delta \mu_y)_{\text{SEXT}} \\
W \times \Delta \nu_x \\
W \times \Delta \nu_y
\end{bmatrix} = [A] \cdot 
\begin{bmatrix}
(\Delta K_1)_{\text{SEXT}} \\
(\Delta K_1)_{\text{QUAD}}
\end{bmatrix}
$$

Two additional constraints on the global tunes help to maintain the same working point on the tune diagram. Two last equations in the system (4) are weighted accordingly.

The procedure can apply to local or global compensation for any given set of IDs. In the case of global compensation one may specify any combination of the ring quadrupoles in the $\Delta K_1$ vector on the right side of (4).

This procedure was tested and implemented as a routine named ID_corr in TRACY-2. Results of the test are shown in Fig. 2 and 3.

![Fig. 2: Eighteen IDs with random field strength are included into the lattice](image)

For the example shown in Fig. 3 the residual beta- and phase-beats not only meet the specifications given for the NSLS-II lattice (Table 1), but are less than by at least an order of magnitude.

![Fig. 3: The same, but after using the algorithm of local compensation](image)

**DYNAMIC APERTURE WITH IDS**

As the linear term in (1) is compensated we may proceed to evaluation of impact of the higher order terms. The driving term for the vertical tune shift with amplitude is given by the following expression [6]:

$$
h_{0020} = \frac{3}{8} \cdot b_2 L \cdot \beta_y^2,
$$

where the ID is represented as an octupole with zero length, $b_2 L$ is integrated octupole component of the ID field, $\beta_y$ is beta-function in the ID center. Driving terms for octupolar modes are given by the following expressions [6]:

$$
h_{0020} = \frac{3}{8} \cdot b_2 L \cdot \beta_y^2 \cdot \exp(2\mu_4) \quad h_{0040} = \frac{3}{8} \cdot b_2 L \cdot \beta_y^2 \cdot \exp(4\mu_4)
$$

Table 2 shows analytic estimates of the driving terms in presence of a single ID in comparison with that of the bare lattice.

<table>
<thead>
<tr>
<th>Lie Generator</th>
<th>Sextupole Scheme</th>
<th>SCU</th>
<th>MGU</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{0020}$</td>
<td>606.9</td>
<td>1254.6</td>
<td>1102.4</td>
<td>818.1</td>
</tr>
<tr>
<td>$h_{0030}$</td>
<td>76.2</td>
<td>41.6</td>
<td>9.5</td>
<td>52.6</td>
</tr>
<tr>
<td>$h_{0040}$</td>
<td>46.6</td>
<td>10.6</td>
<td>4.7</td>
<td>25.6</td>
</tr>
</tbody>
</table>

Analysis of this table leads to an insight on acceptable choice of the IDs for the NSLS-2 lattice. Direct comparison between the table columns for the bare lattice and that with a single ID shows that driving term values, induced by a single ID, are comparable or may even exceed these for a well-compensated ring optics providing a large DA.

As an illustration we compute the DA [4] (Fig. 4) for the bare lattice and for the lattice with a single ID of each kind (Table 2). During this computation we assumed RF turned off and random lattice misalignments of 100 μm (rms) only in quadrupoles. The DA was observed in the middle of the injection straight ($\beta_x \approx 18.5$ m, $\beta_y \approx 3.9$ m).
Analysis of Fig. 5 shows that the IDs listed in Table 2 may be acceptable. Longer insertion devices may present hazard of substantially reducing the DA and should be considered if minimization of the driving terms is possible.

**CONCLUSION**

In this paper we discuss impact of beam dynamics caused by strong-field insertion devices in the state-of-art storage ring light sources. We have developed a method for minimization of the beta- and phase-beat in the ring lattice. Applying this method to the NSLS-2 case we obtain the DA in presence of various IDs. Estimates and computations for a helical ID are in progress.

**ACKNOWLEDGEMENTS**

T. Shaftan is grateful to J.B. Murphy for discussions and comments.

**REFERENCES**


---

*Note, these sextupoles can be treated as independent families.*