COMPARISON OF DOUBLE BEND AND TRIPLE BEND ACHROMATIC LATTICE STRUCTURES FOR NSLS-II *

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Abstract
The Double Bend Achromatic (DBA) and the Triple Bend Achromatic (TBA) lattice have been studied rather extensively for use for the NSLS-II storage ring. The advantage of the TBA compared to the DBA in terms of emittance per period is well known. However, the DBA has the advantage of greater number of ID straight sections for the users and maybe easier to tune the dispersive section for reduced chromatic sextupole strength. We present a comparison of these lattices based on optimization of the non-linear driving terms using high order achromatic cancellation of driving terms of the non-linear lattice.

INTRODUCTION
The National Synchrotron Light Source (NSLS) was one of the first 2nd generation light sources and has operated more than 23 years, with a large user community. The existing storage rings, have been improved over the years, but the demand for more undulator beam lines with higher brilliance, can not be accommodated within the present facility. A proposal to upgrade the facility with a 3rd generation, ultra-high brilliance storage ring has been presented, NSLS-II, the parameters are listed in Ref. [1].

As with most new light sources, we have performed a study of the lattice structure that best meets the design goals for NSLS-II. This paper will describe our attempts to meet these goals with the TBA and DBA lattices.

LINEAR LATTICE DESIGN
The TBA or DBA lattice composed of $N_p$ periods with iso-magnetic field dipoles and $\theta_p = \frac{2\pi}{N_p}$ bend angle per period, have a minimum emittance [2] given by

$$\epsilon_{\text{METBA}} = \frac{C_y \gamma^2}{4\sqrt{15} J_s} \theta_p^3 \approx 40.707$$

or

$$\epsilon_{\text{MEDBA}} = \frac{C_y \gamma^2}{4\sqrt{15} J_s} \theta_p^3 \approx 8$$

where $\gamma$ is the relativistic energy, $J_s$ is the horizontal partition factor and $C_y = 3.84 \times 10^{-13}$ m. At 3GeV a 24 period ring has $\epsilon_{\text{METBA}} \approx 0.38 nm$ or a factor of more than 4 times less than the desired emittance. The DBA lattice, having a factor of ~5 more emittance per period would require a 32 or more period lattice to achieve the same emittance goal.

Figure 1: Lattice functions for 12 period TBA(top) and 15 period DBA(bottom) lattices.

Achieving even close to the minimum emittance in a small circumference ring requires large betatron phase advance and strong quadrupoles. Correcting for the resulting high chromaticity requires strong sextupoles reducing the dynamic apertures (DA) necessary for injection and good lifetime.

The basic TBA and DBA cell structures are presented in Refs. [1&3]. The reason the TBA has reduce emittance results from a large portion of the bend angle coming from the center dipole, the so called minimum emittance dipole. This advantage is only obtained by making a minimum in $\beta_s$ and $\eta_s$ functions at the center dipole.

This condition causes these functions to be small between the dipoles, resulting in strong chromatic sextupole necessary to correct for the large chromaticity of these low emittance lattices. On the other hand the DBA lattice

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gives considerable freedom to maximize these functions in the dispersion region, resulting in lower chromatic sextupole strengths. The peak values of dispersion were: 0.28, 0.45 for the basic TBA and DBA respectively. Both lattices have been similarly optimized with increased bend radius dipoles and ID quadruplets as discussed in other references [1,3]. The TBA had lower chromaticity values \((\xi_{xy}) = (2.29, 1.04)/\text{period} \) but stronger chromatic sextupoles \(\{b3*L(SF,SD)=2.8, -3.2 \, \text{m}^{-2}\} \). The DBA had \((\xi_{xy}) = (3.31, 1.11)/\text{period} \) and sextupole strengths of \(\{b3*L(SF,SD)=2.4, -2.3 \, \text{m}^{-2}\} \). These lattices both had a similar emittance of \(\approx 2.1\, \text{nm} \), which lost the original advantage of the TBA over the DBA.

DYNAMIC APERTURE OPTIMIZATION

Both lattices have had their DA optimized using 3-chromatic sextupoles and 8-geometric sextupoles in the achromatic ID straight sections. The procedure described in [1,4] was used to optimize the lattice working point for maximum cancellation of the sextupole driving terms for a high order achromatic condition. These tunes are listed in Table I. The DA and frequency maps [5] for these optimized lattices are shown in Figure (2 & 3).

Both lattices have achieved adequate control of DA and diffusion, and have maintained their DA when alignment tolerances are included. However, the TBA has better control over higher order chromaticity than the DBA lattice [1,3]. The 2\(^{nd}\) order chromaticity, \(\xi_{(2)} \), has been shown [6, 7] to depend on the momentum dependence (\(\delta\)) of \(\eta\) and \(\beta\) functions and is given as a sum over quadrupoles (\(b2L\)) and sextupoles (\(b3L\)) strengths:

\[
\xi_{(2)} = \frac{1}{2} \sum_{j} \left(2b2L_{j} \frac{\partial \eta_{j,j}}{\partial \beta} - b3L_{j} \frac{\partial \eta_{j,j}}{\partial \delta} \right)
\]

Figure 4: 2\(^{nd}\) order chromaticity contribution for the 2\(^{nd}\) term in Eq.(3), TBA(12x2)\{top\} and BA(15x2)\{bottom\} lattices, horizontal(green) and vertical(red).

The large \(\beta\) and \(\eta\) variations with \(\delta\) of the DBA lattice compared to the TBA contributes to the larger 2\(^{nd}\) and 3\(^{rd}\) order chromaticity. Introducing a 3\(^{rd}\) chromatic sextupole allows \(\xi_{(2)}\) to be reduced [6,7]. Figure (4) compares the
first term inside the braces of Eq.(3) for these two lattices, which shows the DBA lattice contribution ~20X the TBA value. The sextupoles have been tuned to reduce the 2nd order chromaticity, but the biggest effect appears to come from a 3rd order chromaticity that will scale with the 2nd order term similar to that shown in Eq.(3). This chromaticity problem is shown in Figure (5), from a tracking result. The large asymmetry for δ< 0 for the DBA lattice, will reduce the momentum aperture once synchrotron oscillations are included, but is adequate for a proposed 3% RF bucket height and with alignment tolerances.

DAMPING WIGGLERS FOR EMITTANCE CONTROL AND REDUCTION

From the DA control point of view both lattices performed adequately, but once again the increased number of ID straight sections won out for the DBA lattice and will provide a potential for even smaller emittance that couldn’t be achieved with the TBA lattice, due to the reduced DA for lower emittance. With this limitation for the TBA lattice, it was suggested [3] that the additional straight sections of the DBA could be provided with damping wiggles, reducing the emittance. With up to 48m of damping wiggles it was shown this could provide 3-4x reduction of the lattice emittance without the severe nonlinear dynamics issues imposed by retuning the lattice. These wiggles will also have nonlinear issues, but these are less severe than for the small gap and wavelength undulators that will be the primary goal of this storage ring. These issues are addressed in more detail in [3,8].

Damping wigglers have been proposed many years ago[9], but haven’t been used in 3rd generation light sources. Since emittance reduction depends on the ratio of dipole (ρd) to wiggler (ρw) bend radius, ρw was increased to enhance the effect of the wiggler field. If the wiggler field is too high the energy spread of the beam will be large, reducing the brilliance of undulator beams, especially at the higher harmonics. Increasing ρw counters this effect by lowering the natural energy spread of the ring as shown in Table I.

Table I Comparison of DBA and TBA lattices presented.

<table>
<thead>
<tr>
<th>Parameter\Latt.</th>
<th>DBA- (15 x2)</th>
<th>TBA-(12 X 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nux/cell(Total)</td>
<td>1.0783 (32.35)</td>
<td>1.3448 (32.275)</td>
</tr>
<tr>
<td>Nuy/cell(Total)</td>
<td>0.5427 (16.28)</td>
<td>0.6156 (14.775)</td>
</tr>
<tr>
<td>ξx/cell(Total)</td>
<td>-3.395 (-101.9)</td>
<td>-2.085 (-50.03)</td>
</tr>
<tr>
<td>ξy/cell(Total)</td>
<td>-0.944 (-28.03)</td>
<td>-1.145 (-27.49)</td>
</tr>
<tr>
<td>βx (m)</td>
<td>2.81 (17.75)</td>
<td>2.87 (14.6)</td>
</tr>
<tr>
<td>βy (m)</td>
<td>1.25 (8.84)</td>
<td>4.2 (9.67)</td>
</tr>
<tr>
<td>εx (nm)</td>
<td>2.03</td>
<td>2.24</td>
</tr>
<tr>
<td>ρ (m)</td>
<td>25.02</td>
<td>18.33</td>
</tr>
<tr>
<td>ID (# - L [m])</td>
<td>15-5 (15-8)</td>
<td>12-6 (12-8)</td>
</tr>
<tr>
<td>Circum (m)</td>
<td>780.3</td>
<td>758.355</td>
</tr>
<tr>
<td>α1 (*10^-4)</td>
<td>3.68</td>
<td>4.35</td>
</tr>
<tr>
<td>α2 (*10^-3)</td>
<td>4.27</td>
<td>4.00</td>
</tr>
<tr>
<td>δE/E (10^-4)</td>
<td>5.135</td>
<td>6.4</td>
</tr>
<tr>
<td>Jx</td>
<td>0.9982</td>
<td>1.235</td>
</tr>
<tr>
<td>Uo (KeV)</td>
<td>286.37</td>
<td>390.78</td>
</tr>
<tr>
<td>DA (X x Y mm)</td>
<td>(25 x 20)</td>
<td>(24 x 23)</td>
</tr>
<tr>
<td>dP/P</td>
<td>&gt; +4 /- 3.5%</td>
<td>&gt; +/-4 %</td>
</tr>
</tbody>
</table>

CONCLUSIONS

We have reviewed the study performed by the NSLS-II design team which compared TBA(12x2) and DBA(15x2) lattices to meet the design goals and provide the potential for improved operations of the ring. Both lattices performed comparably in DA and alignment tolerances. The TBA was more flexible in tuning and in reducing higher order chromatic effects. However, the DBA, with 6 additional ID’s, has the potential for reducing the emittance by factors of >3.5X with less non-linear dynamics reduction of the DA. The DBA(15x2) has now become the preferred lattice for the NSLS-II project[3].

REFERENCES