TUNING ALGORITHMS FOR THE ILC BEAM DELIVERY SYSTEM*

J. K. Jones, ASTeC Daresbury Laboratory, Warrington, Cheshire, WA4 4AD, UK.

Abstract
Emittance preservation is an important aspect in the design and running of the International Linear Collider (ILC) with a direct consequence on the luminosity of the machine. The Beam Delivery System represents a major problem in this respect as it produces emittance dilution effects that are difficult to correct and that have a direct effect on the emittance as seen at the interaction point, and thus upon the luminosity of the machine. Tuning algorithms for this section of the machine rely on the correction of aberrations through the use of linear and higher order knobs, using correction magnets or movers distributed throughout the system. Alternative systems are also discussed. The design and implementation of these tuning algorithms, and their effectiveness in a variety of cases, are investigated.

INTRODUCTION
The ILC Beam Delivery System (BDS) is a ~2.5km long section of the ILC designed to transport the colliding beam from the end of the linac to the interaction point. Due to its high energy, emittance dilution effects from synchrotron radiation become a problem, and this is only exacerbated by errors in the BDS. The BDS acts as the last correctable section of the machine before collision and so is partially designed to correct dilution effects from upstream of itself (though not all aberrations can be corrected in the BDS). Coupled with a small lever arm, this makes the BDS particularly sensitive to internal error sources. A low energy model similar to the ILC BDS is currently under construction at KEK and called the ATF2. This extension to the current ATF damping ring, will allow studies to be performed on the likely BDS design well before it is finally built. Here, the two systems are used interchangeably. Though results are presented only for the ILC BDS.

All emittance dilution effects seen at the IP, whether caused in the BDS or upstream of it, can be corrected for in several manners: trajectory correction systems minimise the orbit excursions in linear and non-linear magnetic elements, reducing dispersive effects and higher order aberrations; dedicated emittance measurements combined with skew-quadrupole magnets can remove coupling effects coming from the main linac; fast luminosity monitors can be used to feedback position and angle offsets for the final beam collisions.

To achieve maximum luminosity at the IP, however, it would be useful to have a set of dedicated tuneable knobs to remove any final aberrations that these other correction systems can not correct. If these tuneable knobs are placed as close as possible to the IP, their effect is minimal anywhere other than at the IP, and thus we can generally ignore effects from the system in the wider scope of the BDS, simplifying their design.

To create a variety of linear and 2nd order knobs, sextupole magnets positioned in optimal phase relationships, and physically close to the IP, are used. Using 4 degrees of freedom (2 transverse displacements, field strength and rotations around the s-axis) it is possible to create adequate tuning knobs to correct all major linear aberrations, including several coupling terms, and many important 2nd order effects. Correction of 3rd order and higher aberrations requires higher order multipoles.

TUNING KNOB DESIGN
The tuning knobs are designed by generating linear fits of the tuneable aberrations for each degree of freedom. A matrix of these linear gradients can then be inverted, and solved for a given aberration. The method used in this case is Singular Value Decomposition (SVD), due to its inherent robustness.

In the case where the aberration is non-linear in a given degree of freedom, or where it is difficult to produce a highly orthogonal knob using SVD inversion, a genetic algorithm is employed to optimise the orthogonality.

Genetic algorithms allow a relatively rapid exploration of problem space without having specific knowledge of the topology of the solution space. The parameters are optimised against the ratio of the absolute value of the desired aberration versus the sum of all other aberrations at that point. Weighting is used to ensure orthogonality against the dominant emittance diluting aberrations.

In the system explored in this paper, we ignore aberration correction mechanisms upstream of the chosen sextupole magnets, and we do not require new magnets in the design. It may be possible to improve the orthogonality of certain tuning knobs by inclusion of these magnets. One exception to this is the trajectory correction system employed to minimise beam displacements in magnetic elements. The trajectory correction system employed is based on inversion of the orbit response matrices, again using SVD[1]. The actuators for this correction system can be quadrupole...
magnets on transverse movers, or dipole corrector magnets placed near to the quadrupole magnets.

**LINEAR TUNING KNOBS**

There are 4 major linear tuning knobs that are generated for aberrations at the IP. They relate to the 4 important twiss parameters during collisions: $\beta_x$, $\beta_y$, $\eta_x$, and $\eta_y$. The first 3 aberrations can be corrected for using horizontal displacements of the sextupole magnets. Orthogonality is relatively high, though the horizontal dispersion is not as optimal as the beta function knobs. Vertical dispersion correction is performed using vertical offsets of the sextupoles. Vertical displacements have the disadvantage of having a non-linear dependence in the other 3 linear aberrations. Vertical sextupole displacements also cause linear coupling in the beam. Optimisation of the vertical dispersion tuning knob is performed using a genetic algorithm. Relatively good orthogonality out to reasonable values of the vertical dispersion are achievable.

**COUPLING KNOBS**

In addition to the 4 linear knobs, there are also several linear coupling knobs that are created. In order to determine the dominant coupling aberrations as seen at the IP, the BDS was modelled with a variety of errors and the resulting emittances correlated with aberration amplitude. This gives a list of the most important aberrations, and leads to a weighting scheme for determining the required orthogonality of the tuning knobs created. The 4 major coupling aberrations that are minimised are: $y'x$, $yx$, $y'x'$, $yx'$. All 4 of these aberrations are tuned using vertical offsets in the sextupoles. As with the vertical dispersion, orthogonality against the non-linear dependence on other linear aberrations is performed via a genetic algorithm. Relative orthogonality is more difficult than for the vertical dispersion but is achievable to the required degree.

**SECOND-ORDER KNOBS**

To determine the required 2nd-order knobs, a similar mechanism as for the coupling knobs was used; the BDS was modelled under error conditions and the dominant aberrations determined from emittance growth considerations. It was determined that there were 12 relevant 2nd order terms that were important and that could be tuned using one of the 4 degrees of freedom available. 6 of the 2nd-order aberrations are tuneable by sextupole rotations around the S-axis, with the remaining 6 tuneable via sextupole strength variations. This creates an over-constrained problem with only up to a maximum of 5 sextupoles. Half of these aberrations knobs could not be made orthogonal with the matrix inversion technique, and orthogonality of the other knobs was not as good as would be hoped. In this case we ignore tuning of those knobs that were not orthogonal, though we maintain orthogonality conditions against them for the other tuning knobs. It may be possible to produce greater orthogonality using the genetic algorithm described, but this has not been performed as of yet.

**SIMULATION PROCEDURE**

Simulation of tuning knobs was performed with the MAD 8.23DL code, interfaced through the Mathematica based MADInput system [2]. The Mathematica code allows rapid prototyping of the different aberration knobs, and allows a wide variety of optimisation algorithms to be employed.

Correction of the trajectory is via SVD inversion of the trajectory response matrix using dipole correctors. All quadrupole and sextupole magnets are assumed to have beam position monitors next to them, and all quadrupoles have dipole corrector magnets. There is assumed to be a BPM at the IP, though this may in reality be inferred from other effects, such as luminosity. SVD inversion of the response matrix is used due to its inherent robustness and the ability to easily cope with faulty BPMs and other effects. The ability to weight certain BPMs is another advantage not explored here.

The tuning of individual aberrations is performed using a Nelder-Mead simplex 1-D algorithm[3] in Mathematica. The Simplex algorithm is a relatively robust optimisation procedure that can be used in a variety of optimisation scenarios. Other optimisation methods such as the Brent’s root-finding algorithm have also been studied, all of which achieve similar results. For each aberration a starting amplitude needs to be defined to ensure that the algorithm does not start outside of the stable region of problem space. The rapid ability of the simplex algorithm to self-adjust to larger values ensures that stability of the starting system is less important, and conservative starting values can be defined. This remains a limitation for some of the root-finding algorithms, such as the Brent method, where maximum constraints must be set, which may limit the achievable correction on any one iteration.

One limitation of the chosen method is the assumption of 1 parabolic minima. This should be confirmed, though the problem of non-linear stability may make this more difficult.

The chosen procedure has the following steps:

1. Correct the trajectory iteratively 3 times
2. For each aberration
   a. Correct the trajectory
   b. Optimise aberration using simplex algorithm
3. Iterate.

There are 14 tuning knobs that are optimised. In all cases the beam spot size at the IP is taken as the figure of merit, and is assumed to be directly related to the luminosity of the machine. The fitness value for the simplex optimisation is:

$$\text{Fitness} = \sqrt{\frac{x^2}{x^2_0} + 500\frac{y^2}{y^2_0}}$$

where $x$ and $y$ are the beam sizes and the 0 subscript denotes the nominal beam size.
This value will be taken as the “luminosity” in the rest of this paper.

### TEST CASES

Simulations are performed to test the effectiveness of the designed tuning knobs to correct well formed aberrations.

#### Quadrupole Gradient Error

A $\Delta k/k$ error of $10^{-3}$ is applied to quadrupole QD0. This produces horizontal and vertical waist shifts at the IP, as well as an increased horizontal dispersion. Applying 1 iteration of the 3 horizontal motion linear tuning knobs we recover over 99.9% of the luminosity. Sextupole horizontal motion is of the order $40 \mu$m r.m.s.

#### Quadrupole Rotation

Quadrupole QD0 is rotated by 1 mrad around the s-axis. This leads to both a vertical dispersion as well as coupling aberrations. The dominant contribution is from the vertical dispersion, and applying this knob regains over 95% of the luminosity. Including the 4 coupling knobs we get a vertical emittance 99.6% of the nominal luminosity. The beam sizes before and after tuning are illustrated in Figure 1. Sextupole vertical motion is around $50 \mu$m r.m.s.

#### Sextupole Field Error

A $\Delta k_2/k_2$ error of $10^{-2}$ is applied to the final sextupole in the BDS, SD0. This leads to higher order aberrations within the beam at the IP. Use of all 6 of the second order tuning knobs restores >99% of the luminosity in one iteration.

### BDS EMITTANCE TUNING SIMULATIONS

In this simulation the entire BDS is modelled with random errors assigned to all quadrupoles and sextupoles. The error magnitudes are given in Table 1. The errors are Gaussian distributed with a cut at 2sigma. No allowance is taken for correlated magnet motion.

No other tuning is performed except for constant trajectory feedback. The tuning algorithm recovers over 70% of the luminosity on the first iteration, with over 95% after 2 further iterations. A bar chart of the luminosity as a function of aberration correction is shown in Figure 2.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta x$ (µm)</th>
<th>$\Delta y$ (µm)</th>
<th>$\Delta \Psi$ (mrad)</th>
<th>$\Delta K/K$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole</td>
<td>30</td>
<td>30</td>
<td>0.1</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Sextupole</td>
<td>30</td>
<td>30</td>
<td>0.1</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>

Table 2 R.M.S. sextupole changes during tuning.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta x$ (µm)</th>
<th>$\Delta y$ (µm)</th>
<th>$\Delta \Psi$ (mrad)</th>
<th>$\Delta K/K$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sextupole</td>
<td>342</td>
<td>61</td>
<td>0.77</td>
<td>7.4</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

1st and 2nd order tuning knobs have been created for the ILC BDS, and have been shown to work well in a variety of cases. Generally, a method for creating and using these tuning knobs has been implemented in Mathematica, and this provides a foundation for further investigations.

To fully assess the usefulness of these tuning knobs to the real ILC BDS, they should be fully implemented in an integrated simulation environment. SVD matrix inversion to create the tuning knobs could also be further analysed. SVD matrix inversion may allow a more robust set of tuning knobs to be created.

Other methods of tuning the BDS should also be investigated. The use of “dumb” optimisation routines, such as genetic algorithms as well as a method that works on the 6x6 “normal-beam to error-beam” map are under investigation.

### REFERENCES

