A CURE FOR MULTIPASS BEAM BREAKUP IN RECIRCULATING LINACS*

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Abstract
We investigate a method to control the multipass dipole beam breakup instability in a recirculating linac including energy recovery. Effectiveness of an external feedback system for such a goal is shown clearly in a simplified model. We also verify the theoretical result with a simulation study.

INTRODUCTION
Following the successful demonstration of energy recovery in a recirculating linac at the Jefferson Laboratory several high current - high beam power accelerator projects based on energy recovering linac technology are presently under intensive studies worldwide. Typically average beam currents for these future projects are very high in the neighborhood of a few100 mA. Being a multipass machine a recirculating linac suffers from a regenerative type instability known as multipass beam breakups not existing in a traditional linac. Multipass dipole beam breakup phenomenon[1,2] has been investigated quite extensively in the design stage of the 5-pass CEBAF superconducting linac. An efficient damping of dangerous dipole HOMs is a critical issue in cavity design and especially so for a SRF cavity. If the beam breakup threshold is still too low even after best effort damping, it is time to ask whether the breakup can be controlled with an external feedback. In this note we present an analysis on how one might control this kind of beam breakup with the help of a feedback system.

A BEAM BREAKUP ANALYSIS
When a bunched beam with a dipole moment passes through a cavity, dipole modes of the cavity are excited and act as dipole magnets depending on the passing beam. Consequently, the beam on the 1st pass gets perturbed by the dipole wakes excited by all preceding bunches, and dipole kicks transported back to the cavity induce dipole moments, which in turn add to the existing dipole modes on its 2nd pass. Thereby a feedback loop is closed, an exponential increase of the wakes is possible when the beam current exceeds a threshold. When there is an external feedback system which can provide an additional modulation to the induced dipole moments, a destructive interference can be initiated under a favorable phase relationship stopping the growth of a beam breakup.

Let us begin by assuming that the whole accelerator is consisted of a single cavity and the bunch sees the cavity only twice. We also assume that the beam will be offset only in the x-z plane and consider a case with no feedback first.

We start by noting that the integrated dipole wake potential $G_1(t)$ is obtained from the following well known relation,

$$G_1(t) = \int_{-\infty}^{t} W_1(t-t')D(t')dt'$$

where $W_1(t)$ is the delta functional dipole wake function and $D(t)$ is the x component dipole moment of the beam bunch when the bunch passes the cavity at time $t$.

For a single dipole mode with an angular frequency $\omega_a$ the wake $W_1(t)$ is given by

$$W_1(t) = 2\frac{ck_{loss}}{\omega_a} \exp(-\frac{\omega_a}{2Q}) \sin\omega_a t$$

for $t > 0$, and $W_1(t)$ is equal to 0 for $t < 0$ by causality. Here $k_{loss}$ is the usual loss factor for the dipole mode defined as follows,

$$k_{loss} = \frac{\omega R}{4Qb^2}$$

where $b$ is the beam pipe radius and $R/Q$ is to be evaluated at $b$.

A bunch entering the cavity with an accumulated wake excited by all preceded bunches will get a momentum kick,

$$\Delta p_x = Ne^2 \frac{G_1(t)}{c}$$

where $N$ is the total number of electrons in the single bunch.

This momentum kick will transform to a dipole moment

$$D(t) = \frac{N r_0^2}{\gamma} M_{1z} G_1(t-t_0) \sum \delta(t-t_0-mt_0)$$

at the cavity on its 2nd pass where $r_0$ is the classical radius of the electron, $r_0 = 2.818 \times 10^{-13}$ cm, and $t_0$ is the recirculation path length in time and $t_0$ is the fundamental bunching period and the summation $\sum$ is over all integer values of m. $\delta(t)$ is the
Dirac delta function. $M_{12}$ is the angular-to-position component of the recirculation matrix from the cavity to the cavity.

Inserting this induced dipole moment back into the equation for the integrated wake function we obtain a homogeneous integral equation for $G_1(t)$,

$$G_1(t) = \frac{N_{t_0}}{\gamma} M_{12}$$

$$\times \int W_1(t - t') G_1(t' - t_\kappa) \sum_m \delta(t' - t_\kappa - mt_0) dt'$$

Searching for an exponentially increasing solution for $G_1(t)$ we find the following expression for the threshold current, $I_{th}$

$$I_{th} = \frac{-2\omega E \exp(-\frac{\alpha t_\kappa}{2Q})}{ec(\frac{R}{Qb_1^2})QM_{12} \sin \alpha t_\kappa}$$

where the beam energy $E = \gamma m_e c^2$.

Next we consider a case with an external feedback system. We picture a system with a pick-up positioned somewhere in the recirculation path and the kick is provided to the beam at a location somewhere downstream of the pick-up. Now when there is such a feedback system, the dipole moment at the cavity on its 2nd pass will be different from the previous expression. We get

$$D(t) = \frac{N_{t_0}}{\gamma} \left\{ M_{12} G_1(t - t_\kappa) + \alpha M_{12}^1 M_{12}^2 G(t - t_\kappa - t_\kappa) \right\} \sum_m \delta(t - t_\kappa - mt_0)$$

Here $M'_{12}$ is the (1,2) element of the transport matrix from the cavity to the pick-up point and $M'_{2,1}$ is the (1,2) element of the transport matrix from the kicker to the cavity. And $t_\kappa$ is the time delay or advance depending on the system algorithm being selected and $\alpha$ is a feedback coefficient.

The integral equation for $G_1(t)$ is now given by

$$G_1(t) = \frac{N_{t_0}}{\gamma} \int W_1(t - t') \left\{ M_{12} G_1(t' - t_\kappa) + \alpha M_{12}^1 M_{12}^2 G(t' - t_\kappa - t_\kappa) \right\} \sum_m \delta(t' - t_\kappa - mt_0) dt'$$

The first case is when $t_\kappa$ is equal to zero, which is easy to implement in recirculating linacs (typically they resemble a racetrack). In this case the beam breakup threshold expression is exactly same as before except that $M_{12}$ is replaced by $M_{12} + \alpha M'_{12} M'_{2,1}$. This means that a properly chosen $\alpha$ can reverse the growth of beam breakup effectively raising the threshold of the breakup. In the framework of the simple model studied here it will be an easy task to calculate a range of suitable $\alpha$ value once the machine optics is known. In reality this may have to be determined by measurements.

Now in the case of $t_\kappa$ being different from zero we find the following expression for the threshold current, $I_{th}$

$$I_{th} = \frac{-2\omega E \exp(-\frac{\alpha (t_\kappa + t_\alpha)}{2Q})}{ec(\frac{R}{Qb_1^2})QT_{12} \sin \alpha (t_\kappa + t_\alpha)}$$

Here $T_{12}$ and $t_\alpha$ are to be defined from

$$T_{12} \exp(-it_\alpha) = M_{12} + \alpha M_{12}^1 M_{12}^2 \exp(\frac{\alpha}{2Q} + i\alpha t_\alpha)$$

From the threshold formula one can see clearly how a feedback system can control beam breakups by effectively changing recirculation path length and/or relevant transport matrix for the recirculation.

**A SIMULATION STUDY**

To demonstrate the working of an external feedback system for controlling the multipass beam breakup we have modified one of JLAB’s beam breakup simulation codes, TDBBU[3,4], to accommodate a feedback system with $t_\kappa = 0$ studied in the previous section. Figure 1 shows bunch offsets with a feedback system turned off when the beam current is above the threshold for the machine. The total beam running time was 50 $\mu$s. Figure 2 with the feedback system turned on at $t = 20 \mu$s. When the feedback system was turned on almost instantly with the beam injection, we found a stable beam at all times. Figure 3 shows that. For this particular instance $\alpha = 0.002$.

We list here the machine parameters which were used for the simulation study reported. They are,

- Injection Energy = 5 MeV
- Final Energy = 85 MeV
- Energy Gain = 40 MeV
- HOM frequency = 1890 MHz
- Q = 32000
- Recirculation path length = 61.4 meter

In Figure 4 we show beam breakup threshold versus HOM frequency for various bunching frequencies for the same machine except that Q is now set to 10^6. One should get an appreciation of a range of inherent fluctuation to be expected when determining the dipole beam breakup
threshold and at the same time of the degree of improvement in the threshold one should expect with an external feedback for a given machine. Figure 4 has been generated with MATBBU [2,4].

CONCLUSIONS
The successful demonstration of energy recovery in a recirculating linac with superconducting cavities at the Jefferson Lab FEL[5] has popularized a recirculating linac with energy recovery. Several ambitious electron accelerator projects have recently been proposed to build either a light source or a collider based on this technology. All use superconducting cavities and intend to run with very high average beam current in the neighborhood of a few 100 mA. At such high currents it is conceivable that one of these multipass beam breakups will probably become the limiting factor of the accelerator performance. In this paper we have shown that a feedback system might be effectively used to improve the situation significantly, if one still has to deal with beam breakup problems even after having done one’s best for designing HOM couplers, etc.

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REFERENCES