UPGRADED SYMPLECTIC 3D BEAM TRACKING
OF THE J-PARC 3 GEV RCS

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Abstract

The J-PARC 3 GeV ring is a rapid cycling synchrotron which consists of the large bore magnets. The beam tracking with the 3D distributed magnetic fields is kept developing in order to investigate the beam injection process. In order to improve the tracking accuracy and to save the calculation time, the symplectic integrator with the fractal decomposition method has been introduced. The updated simulation results of the beam injection on the J-PARC 3 GeV RCS and the improved performance of ‘Generic-Solver’ are presented in this paper. The quadrupole fields are also treated as the 3D distributed magnetic fields because they interfere with the bump magnet fields. The remarkable features on the large bore magnet system in the ring accelerator are also discussed.

SYMPLECTIC INTEGRATOR

A beam injection of the J-PARC 3 GeV RCS has been investigated with TRACY-II simulator by solving an equation of motion with a Lorentz force by the Runge-Kutta integration method[1][2]. However, it doesn’t insure the symplecticity of the particle dynamics. A symplectic integrator has been introduced in order to conserve the total energy of system. In addition, it is convenient to include a space-charge force.

The relativistic single-particle Hamiltonian in the electromagnetic fields[3] is given by

\[ H(\vec{q}, \vec{p}, t) = c\sqrt{(\vec{p} - e\vec{A})^2 + m_0^2c^2 + e\phi} \quad (1) \]

where a scalar potential \( \phi \) corresponds to a space-charge force without an external electrical field. The magnetic field \( B(x, y, z) \) is given by the 3D-vector set distributed on a lattice structure. The time step \( \Delta t \) is set to the 1/5 of the minimum time while a particle passes through a cell. Though the magnetic field is a function of the location, it can be approximated to a constant vector during the time step \( \Delta t \). In addition, the \( \gamma \) doesn’t change its value in the magnetic field. In these conditions, the time-evolution operator of the magnetic field \( \exp(\Delta t D_M) \) can be given by simple matrices. When the Hamiltonian \( H \) can be written in a sum of two terms as: \( H = H_M + H_U \), the corresponding time-evolution operator can also be divided to two components, such as:

\[ \exp(\Delta t D_H) = \exp(\Delta t (D_M + D_V)) \quad (2) \]

The exponential operator product can be decomposed by using the fractal decomposition method[4] given in Eq. 3 and Eq. 4.

\[ \exp(t(A + B)) = \prod_{i=1}^{r} \exp(a_i t A) \exp(b_i t B) + O(t^{m+1}) = S_m(t) + O(t^{m+1}) \quad (3) \]

\[ S_{2m}(t) = S_{2m-1}(t) = S_{2m-2}(p_m t) S_{2m-2}(1 - 4p_m t) S_{2m-2}^2(p_m t) \quad (4) \]

where \( p_m = 1/(4 - 2m^{-1/3}) \). Adopting \( m = 2 \), the 4-th order symplectic integrator with space-charge force is obtained. This integrator has been installed into the Generic-Solver which is a sub-program of TRACY-II.

MAGNETIC FIELDS OF THE 3 GEV RCS

The recent design of the beam-injection system of the 3 GeV RCS is shown in Fig. 1, which is installed in a straight section. Because the beam tracking by the GenericSolver is carried out using the Descartes coordinate system, the \( z \) axis is identical to the \( s \) axis in this section. Two bump systems are prepared. One is the shift bump system to form an orbit offset of \( x = 90 \) mm at the carbon stripping foil on the charge-exchange injection, and the other is the paint bump system for beam painting. The shift-bump system will be fully excited during the whole injection period. On the other hand, the paint bump has a time dependency in order to paint up the RCS beam emittance of 216 \( \pi \) mm mrad with the LINAC beam of 6 \( \pi \) mm mrad. The injection process continues for about 320 turns. The carbon stripping foil is placed between SB2 and SB3. The foil position is defined as the injection point.

Correction for the Bump Magnets

The shift-bump magnets SB1 to SB4 are connected in series in order to form a shift-bump orbit. They are the identical magnets, and are required to excite the same strength magnetic fields. The \( Bl \) values obtained from the 3D magnetic field data are listed in Table 1. The \( Bl \) value is an integration of the \( B_y \) field along the \( z \)-axis.

Due to fringe interference, the \( Bl \) balance in the four bump magnets is broken, which causes a closed-orbit distortion, as shown in Fig. 2. Especially, the effective \( Bl \)

<table>
<thead>
<tr>
<th>Name</th>
<th>( Bl ) [Tm]</th>
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<tbody>
<tr>
<td>SB1</td>
<td>0.17373</td>
<td>SB2</td>
<td>0.17548</td>
</tr>
<tr>
<td>SB3</td>
<td>0.17551</td>
<td>SB4</td>
<td>0.17365</td>
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Table 1: \( Bl \) values of the shift bump magnets at the center position: \( x = y = 0 \). Design value = 0.17559 Tm
Correction for the Quadrupole Magnet

Similarly to the dipole fields, quadrupole fields also deviate from the design value due to the field interference. In the case of quadrupole fields, the field deviation appears on a field gradient. The $Gl$ value is defined as an integration of the field gradient along the $z$-axis. The focusing gradient deviation mainly causes the horizontal tune shift, and the defocusing gradient deviation mainly causes the vertical tune shift. A deviation of the field gradient is sometimes more harmful than the dipole field deviation. The field gradient was corrected by using the designed $Gl$ values. Fig. 3 shows the reference particle motions in phase space. In order to see the trajectory, the particle positions for every turn are joined by linear lines. There is no tune difference with respect to the field-mesh size along the $z$-axis. The betatron tune shift, which corresponds to an amount of about 1% of field gradient, is observed in vertical phase space, compared to the design matrix, even after field gradient normalization. The origin of this deviation, however, comes from the changes in the magnet effective length. The effective length of the large-bore magnet extends, and the drift spaces on both sides of the quadrupole magnet are shortened.

In the case of quadrupoles, the field deviation appears on a field gradient. The $Gl$ correction is required in order to cancel the $Bl$ unbalance, which is a mechanical adjustment on the actual magnets. In a calculation, the amounts of the magnetic fields were normalized by using the design $Bl$ value in the GenericSolver program. The red line in Fig. 2 shows the bump orbit after $Bl$ normalization. The residual COD can be suppressed. Similarly, the paint bump magnet fields are also corrected. The resulting bump orbit quality is summarized in Table 2.

<table>
<thead>
<tr>
<th>$x$ (design) [mm]</th>
<th>$x'$ (design) [mrad]</th>
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<tr>
<td>shift 90.26 (90.00)</td>
<td>-0.05 ( 0.00)</td>
</tr>
<tr>
<td>paint 41.09 (41.00)</td>
<td>-5.51 (-5.50)</td>
</tr>
<tr>
<td>all 131.44 (131.00)</td>
<td>-5.51 (-5.50)</td>
</tr>
</tbody>
</table>

values of SB1 and SB4 are about 1% smaller because the field tail becomes short, since the magnetic flux is diverted due to the existence of the quadrupole magnet core. The $Bl$ correction is required in order to cancel the $Bl$ unbalance, which is a mechanical adjustment on the actual magnets. In a calculation, the amounts of the magnetic fields were normalized by using the design $Bl$ value in the GenericSolver program. The red line in Fig. 2 shows the bump orbit after $Bl$ normalization. The residual COD can be suppressed. Similarly, the paint bump magnet fields are also corrected. The resulting bump orbit quality is summarized in Table 2.

In the J-PARC, the linac beam is collimated to $4 \pi$ mm mrad by the collimator at the linac-to-RCS transport line. The beam emittance at the primary foil, however, is estimated to be $6 \pi$ mm mrad at most if the beam blowup occurred by the space charge force after the collimator. At the beam injection line, beam passes through several kinds of time-varying fields until it reaches the primary stripping

TRACKING RESULTS

In the J-PARC, the linac beam is collimated to $4 \pi$ mm mrad by the collimator at the linac-to-RCS transport line. The beam emittance at the primary foil, however, is estimated to be $6 \pi$ mm mrad at most if the beam blowup occurred by the space charge force after the collimator. At the beam injection line, beam passes through several kinds of time-varying fields until it reaches the primary stripping
Figure 3: Particle trajectory in vertical phase space; rev1 corresponds to $\Delta z = 100$ mm mesh and rev2 corresponds to $\Delta z = 25$ mm mesh.

Figure 4: Phase-space distribution of the L3BT beam of $13\pi$ mm mrad in the horizontal plane at the injection point ($s = 82.113$ m) during 308 turns with painting the process. The 1st(black), 101st(red), 201st(green) and 301st(blue) injected bunches are traced.

Figure 5: Phase-space distribution of the L3BT beam of $13\pi$ mm mrad in the vertical plane at the injection point ($s = 82.113$ m) during 308 turns with painting the process. The 1st(black), 101st(red), 201st(green) and 301st(blue) injected bunches are traced.

CONCLUSION

The 3D particle tracking simulation including the realistic fringe field is available by the TRACY-II simulator, which can take into account the 3D magnetic field distributions given by the magnet design code, TOSCA. The TRACY-II was applied to the design of the injection-straight section of the 3 GeV RCS of the J-PARC project, calculating the beam profile distribution of the painted beam. In this process, the followings can be noted:

- Introducing a symplectic integrator, the system phase volume is insured to be conserved.
- A symplectic integrator is more than 5 times faster compared to the Runge-Kutta integrator without space-charge contribution.
- Fractal decomposition method makes it easy to construct a symplectic integrator of an arbitrary order.
- There is no obvious difference in the tracking results of 3 GeV RCS injection process between two methods: Runge-Kutta integration and Symplectic integration.
- The effect from the space-charge force is not large with this simulation.
- Magnetic field interference between the closely located magnets breaks the field balance.
- A long fringe field of the large-aperture quadrupole magnets causes a betatron tune shift.

REFERENCES