A METHOD TO MEASURE THE FOCUSING PROPERTIES (R_MATRIX) OF A MAGNET*

C-AD Dept., BNL Upton, NY 11973

INTRODUCTION

The common method, which is applied in the magnet division of the Brookhaven National Laboratory (BNL), to determine the focusing properties of a magnet, is to measure the magnetic field of the magnet by using various techniques[5]. The magnetic field measurements provide an accurate method to determine the focusing properties of a magnet like dipoles or higher order multipoles. The magnetic-field measurement method however is not very accurate when applied to measure the focusing properties of a magnet like a Siberian Snake[3,4]. In this paper we study the possibility to measure the focusing properties of a Helical Snake[3,4]. In this paper we study the possibility to provide an accurate method to determine the focusing properties of a magnet, is to measure the magnetic field of the magnet, which may be obtained from the BNL Tandem accelerator.

In brief the method consists on, injecting “narrow_beamlets” of heavy ions into a magnet and measuring the coordinates; of these “narrow_beamlets”, at the entrance and exit of the magnet.

From the measurement of the coordinates of the “narrow_beamlets” we can deduce information on the first order transfer matrix elements (R matrix) and higher order matrix elements that define the focusing properties of the magnet.

The focusing properties of a magnet are considered known when the coordinates of a particle at the exit of a magnet can be determined, assuming that the coordinates of the particle at the entrance of the magnet are known. The above sentence can be expressed either, schematically in Figure 1, which shows a magnet with the coordinates of a particle at the entrance and exit coordinate systems, or mathematically in equation (1) below.

\[
x_i(\text{out})=\Sigma R_{ij}x_j(\text{in}) + \Sigma W_{ijl}x_k(\text{in})x_m(\text{in})x_l(\text{in}) + \Sigma \Sigma \Sigma T_{lmno}x_m(\text{in})x_n(\text{in})x_o(\text{in}) + \ldots \text{HOT} \quad (1)
\]

The notation in equation (1) is:

\((x, x', y, y', \delta l, \delta p)\)

Where \(x, y, x', y'\) are the lateral (x, y) and angular (x', y') deviations of the particle from the trajectory of the central particle. The quantity \(\delta p\) is the momentum deviation of the particle’s momentum \(p\), from the momentum \(p_0\) of the central-particle, and \(\delta l\) is the path length difference of the particle’s path from the path of the central particle.

**Figure 1.** Schematic diagram of the magnet with the entrance end exit coordinate system. The beginning of the red arrows show the location of the particle and the direction of the arrow shows the direction of the particle at the entrance and exit coordinate systems.

The coefficients in equation (1) are defined as the partial derivatives of the output coordinates with respect to the input coordinates.

\[ R_{ij} = \frac{\partial x_i(\text{out})}{\partial x_j(\text{in})} \quad 1^{\text{st}} \text{order} \]

\[ W_{ijl} = \frac{\partial x_i(\text{out})}{\partial (\partial x_j(\text{in})\partial x_k(\text{in}))} \quad 2^{\text{nd}} \text{order} \]

\[ T_{lmno} = \frac{\partial x_i(\text{out})}{\partial (\partial x_m(\text{in})\partial x_n(\text{in})\partial x_o(\text{in}))} \quad 3^{\text{rd}} \text{order} \]

\[ \ldots \]

\[ T_{lmno...p} = \frac{\partial x_i(\text{out})}{\partial (\partial x_m(\text{in})\partial x_n(\text{in})\ldots \partial x_p)} \quad n^{\text{th}} \text{order} \]

The knowledge of the coefficients \(R_{ij}, W_{ijl}, T_{lmno}\), and of the higher order coefficients, determine the focusing properties of any magnet.

In the “paraxial ray approximation”, which assumes, that the momentum deviation \(\delta p\) of the momentum \(p\) of the

---

* Work performed under Contract # DE-AC02-98CH10886 with the auspices of the US Department of Energy.

† tsoupas@bnl.gov
The radius of curvature \( \rho \) is defined in the equation

\[ B \cdot \rho = k \cdot q \cdot p / q \]  

(B is the field of the magnet, \( p \) is the momentum, and \( q \) is the charge state of the ion, \( k \) is a constant which depends on the units),

The size of the holes depends on the magnet to be measured. For the Partial Snake magnet the diameter of the holes should be no greater than 0.5 mm.

Figure 2. Schematic diagram of the magnet with the various devices which define the position \((x, y)\) and direction \((x', y')\) of the rays at the entrance and exit of the magnet.

The second device is the “2nd ion-position measuring device” which is located at a specified distance, just after the fringe field of the magnet, consists of a light emitting foil (visual Flag) and a CCD camera connected to a computer. The light emitting foil is a compound of Gadolinium Oxy-Sulfide doped with Terbium (Gd2O2S:Tb). The compound is bonded on an Al foil. The ions emerging from the pin holes of the “1st ion-position defining device” will generate light on the foils of the 2nd or 3rd measuring devices. The emitted light will be detected by the CCD camera and the computer will calculate the location of each “pin-hole image” on the visual flags (foils).

A schematic diagram of the detection system which utilizes a visual flag is shown in Fig. 3. The direction of the rays \((x', y')_\text{in} \) at the entrance of the magnet can be determined from the coordinates \((x, y)_\text{in} \) of the rays at the entrance, as defined by the “1st ion-position defining device”, and the coordinates \((x, y)_\text{out} \) of the rays at the exit as measured by the “2nd ion-position measuring device” when the field of the magnet is off.

Figure 3. Schematic diagram of an ion-position measuring system to be used in the position measurement of the pencil-like ion beams which will enter and exit the magnet.

The third device “3rd ion-position measuring device” is located at the exit of the magnet at a specified distance, from the location of “2nd ion-position measuring device” otherwise it is identical to the “2nd ion-position.
measuring device”. Position measurements of the ions taken from the 2nd and 3rd “ion-position measuring devices” will determine the direction of the ions at the exit of the magnet (x′,y′)out.

The fourth device is the “ray-direction defining” magnet which is a dipole magnet that can change the direction (x′,y′)in of the rays at the entrance of the magnet and also serves to select the charge-state to be used when using heavy ions.

COMPUTER SIMULATIONS

The method to measure the focusing properties of a magnet was tested using computer simulations on a 3D model of the warm helical snake [3] which is installed in AGS. The raytracing into the magnet was performed with the computer code SPRAY [6]. The required 3D magnetic fields of the magnet were computed using the computer code opera [7] on a 3D model of the magnet [4]. Error analysis of the proposed method as well as comparison of the method with the magnetic measurements method has been performed and details appear in [4]. The error analysis showed that the accuracy of the measurement of the beam centroids generated by the “beamlets” should be ±0.025 cm in order to obtain matrix elements of the same accuracy as we can obtain from the magnetic field measurements. We have assumed that the magnetic measurements are performed with a Hall probe which measures the field components at a given point with a relative error of 10⁻³. Zero position error has been assumed in the measurement of the Hall probe.

SUMMARY

A method to determine experimentally the focusing properties of a magnet has been discussed. This method determines the first order matrix elements (R_matrix) of a magnet and the most “significant” higher order matrix elements. The error in the determination of the first order matrix elements is “reasonably small” to provide accurate measurements of the R_matrix of of a magnet.

REFERENCES

[7] Vector Field Inc.