INCOHERENT TRANSVERSE TUNE SHIFTS
DUE TO RESISTIVE LOW-GAP CHAMBERS*

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Abstract
Betatron tune shifts with current, observed at the ESRF and more recently at BESSY, are analysed and compared. The tune shift is supposed to arise from the wake field in the low-gap vacuum chambers for insertion devices. It is found that the same picture applies to both cases, despite different chamber materials and machine parameters.

1 INTRODUCTION
The betatron tune shift with current, observed at the ESRF was identified to be an incoherent effect, induced by the wake field generated in the low-gap vacuum chambers having finite resistivity and asymmetric cross sections [1]. A similar effect was observed earlier in PEP II, which was analysed by S. Heifets upon the same view [2]. S. Heifets pointed out the importance of multi-turn effect, owing to the long range nature of the wake field, as well as diffusion of the magnetic field in the chamber wall that effectively determines the duration time of the wake field in the chamber.

Figures 1: Upper: ESRF 10 mm chamber (internal = 8 mm). Lower: BESSY 11 mm chamber (internal = 11 mm).

In analysing the observations at the ESRF, a difficulty was encountered in explaining the comparable amount of tune shift in both planes, when the vertical shift is expected to be much smaller due to machine optics. A question arose if the diffusion time varies according to the transverse direction. The concerned effect was also predicted to affect seriously the single bunch, which matched well with the observations at the ESRF.

Upon a similar effect recently observed at BESSY as well, a collaboration was made between the two institutes to investigate the concerned effect, which provides an excellent opportunity to verify the findings at the ESRF.

2 METHOD OF ANALYSIS
A code was developed to compute the transverse wake field according to the formulation in Ref. 2, which takes well account of the fact that the wake force felt by the trailing (witness) particle depends explicitly on its own position when the chamber cross section has no circular symmetry. The field was calculated for the chambers in the two machines (Figs. 1) that are supposed to give major contributions (Table 1). The 50 µm copper coating on the vertical surface of the ESRF 10 mm was not taken into account in the present study, on the ground that we are concerned with long range fields.

<table>
<thead>
<tr>
<th>Material No</th>
<th>Material</th>
<th>$dW_y/dy_w$ [VC/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESRF 10 mm</td>
<td>5.95×10^{12}</td>
<td>SS+Cu coating</td>
</tr>
<tr>
<td>ESRF 15 mm</td>
<td>2.29×10^{12}</td>
<td>SS</td>
</tr>
<tr>
<td>BESSY 11 mm</td>
<td>4.45×10^{11}</td>
<td>Al</td>
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</tbody>
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Table 1: Calculated quadrupolar wake fields for the considered low gap chambers of the two machines.

The incoherent tunes of the individual bunches were computed on the basis of the above wake fields with an optics code developed for this purpose. The computation takes into account the low-gap chambers as how they are actually installed in the ring. The focusing strength of a low-gap chamber felt by a given bunch is computed by taking into account the beam filling as well as the multi-turn effect. The code also takes into account the self force a bunch. Namely, the wake force is averaged over the particle distribution in a bunch to deduce an effective focusing strength felt by a single bunch.

3 IMPACT ON MULTIBUNCH
The common observations in the two machines are that the tune shift is positive horizontally, negative vertically, and are comparable in magnitude. In case of BESSY (Fig. 2), the measured tune shift was actually larger vertically.
by ~30%. However, it turned out that there was a non-negligible contribution of the single bunch coherent detuning, which was measured to be roughly \(-8\times10^{-4}/\text{mA}\). Subtracting the latter, the tune shifts become nearly comparable (a curve with diamonds in Fig. 2). The same is not true for the ESRF machine, since as it operates with large chromaticities, the coherent mode detuning is strongly suppressed.

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The comparable incoherent tune shifts raise a puzzle, since if it were actually an effective quadrupole that appears in the machine, the tune shift should be proportional to the beta values at the quadrupole location. As both machines have high horizontal betas in half of the straight sections while vertical betas are always low, one would expect that the tune shift is much larger in the horizontal plane. In applying the argument of S. Heifets to introduce the diffusion time of the magnetic field in the chamber wall, which truncates the multi-turn build up of the wake field, it follows that the diffusion time is effectively longer in the vertical plane to reproduce the comparable tune shift.

Leaving it a later stage to investigate the validity of the deduced picture, let us see here what consequence it gives (Figs. 3). Namely, we introduce two parameters \(l_h\) and \(l_v\), which respectively represent horizontal and vertical turn number at which, the multi-turn build up is truncated. While for the ESRF, it turns out that \(l_h=40\) and \(l_v=600\), for BESSY, they are much larger being \(l_h=1400\) and \(l_v=20000\). Although the parameters for the latter should at least be larger by a factor of three to represent the same diffusion time, owing to its smaller circumference, they turn out to be additionally larger by nearly a factor of 10.

We see that a much longer diffusion time fitted for BESSY is reasonable, on account of the much better conductivity of aluminium used. One can intuitively interpret that it takes a longer time for the magnetic field to diffuse in a material which is more difficult to penetrate. In fact, Heifets assumes the diffusion time to be inversely proportional to the resistivity, which by itself would give a factor of 26 in place of 10 above [2]. It is also interesting to notice that despite the large difference, the ratio \(l_v/l_h\) happens to be nearly equal between the two machines (~15). In view of the degree of freedom on the beta values, whose ratios are not equal between the two machines, as well as of the similarity of the chamber cross section (Figs. 1), the closeness of \(l_v/l_h\) may not be accidental.

While the average tune shift involves ambiguity associated with the multi-turn effect, the tune shift between head and tail in a bunch train depends quasi solely on the single turn effect. On the basis of this fact, the head-to-tail tune shift was measured in both machines in 1/3 filling. It turned out that the only case where non-zero values were systematically measured was the horizontal plane in the ESRF machine. Including which, the measured results were all in basic accordance with the expectation (Fig. 4).

![Figure 2: Measured incoherent tune shift in BESSY.](image)

![Figure 3: Deduction of diffusion parameters to reproduce the measured incoherent tune shifts.](image)

![Figure 4: Measured horizontal tune shift between head and tail of a bunch train (1/3 filling) in the ESRF.](image)
4 IMPACT ON SINGLE BUNCH

The model predicts a large horizontal incoherent tune shift in single bunch, due to the combination of a strong short range force of the resistive-wall wake field and a higher current per bunch. Here, no contribution is expected from multi turns. As compared to multibunch, the tune shift per beam current is supposed to be 20 times larger for the ESRF, while merely 3 times for BESSY. In case of single bunch, it is difficult to distinguish the incoherent tune shift from the observed head-tail mode frequency shifts. However, it is interesting to observe that, for both machines at zero chromaticity, modes ±1 are focused, while mode 0 stays quasi constant (Figs. 5). The observed tune shifts are in a comparable range as compared to expected. It may be that the constant behaviour of mode 0 is a result of cancellation between the usual defocusing of the broadband impedance and the incoherent tune shift.

At the ESRF, the machine operation is actually disturbed seriously by the supposed single bunch incoherent tune shift. The horizontal threshold of the transverse mode coupling instability, in which mode −1 approaches mode 0 as contrary to the vertical case, is continuously descending as more low-gap chambers are installed, resulting in becoming as low as the vertical threshold. Not only, but another marked finding was the ratio of the two diffusion times being nearly equal in the two machines, which might have a physical meaning. In all cases, the measured head-to-tail tune shifts were in agreement with the expectations. It was also interesting to observe that, in single bunch, head-tail modes were focused horizontally in both machines, in accordance with the expectation. The amount of tune shifts was also in the expected range.

5 CONCLUSION

Due to the use of aluminium for the low-gap chambers in BESSY, the wake field was nearly one order of magnitude smaller than those of the ESRF using stainless steel. Nevertheless, the tune shifts were observed in BESSY in the multibunch filling, which was interpreted as a much longer diffusion time for the corresponding machine. This, however, was indeed what is expected for the magnetic field diffusion with lower resistivity. An equally interesting finding was the comparable magnitude of tune shifts in both machines, when the vertical is always expected to be smaller. This led us to interpret the diffusion time to be longer vertically. Not only, but another marked finding was the ratio of the two diffusion times being nearly equal in the two machines, which might have a physical meaning. In all cases, the measured head-to-tail tune shifts were in agreement with the expectations. It was also interesting to observe that, in single bunch, head-tail modes were focused horizontally in both machines, in accordance with the expectation. The amount of tune shifts was also in the expected range.

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7 REFERENCES