ACCELERATOR PHYSICS CHALLENGES OF X-RAY FEL SASE SOURCES*

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Abstract
A great deal of international interest has recently focused on the design and construction of free-electron lasers (FEL) operating in the x-ray region (~1 Å). At present, a linac-based machine utilizing the principle of self-amplified spontaneous emission (SASE) appears to be the most promising approach. This new class of FEL achieves lasing in a single pass of a high brightness electron beam through a long undulator. The requirements on electron beam quality become more demanding as the FEL radiation wavelength decreases, with the 1-Å goal still 3-orders of magnitude below the shortest wavelength operational SASE FEL (TTF-FEL at DESY [1]). The sub-picosecond bunch length drives damaging effects such as coherent synchrotron radiation, and undulator vacuum chamber wakefields. Unlike linear colliders, beam brightness needs to be maintained only over a small ‘slice’ of the bunch length, so the concepts of bunch-integrated emittance and energy spread are less relevant than their high-frequency (or ‘time-sliced’) counterparts, also adding a challenge to phase space diagnostics. Some of the challenges associated with the generation, preservation, measurement, and stability of high-brightness FEL electron beams are discussed here.

1 INTRODUCTION

The promise of x-ray SASE FELs, as compared with 3rd generation light sources, is nearly ten orders of magnitude increase in peak photon brightness and two orders of magnitude reduction of pulse length. This remarkable step in performance is made possible with the advent of the FEL slippage length, which is the electron-to-photon contrast to linear colliders, where particle collisions multiply by the radiation wavelength. For 1-Å radiation SASE saturation, and is the number of undulator periods multiplied by the radiation wavelength. For 1-Å radiation and typically several thousand undulator periods of a few centimeters, the slippage length is less than one micron. This corresponds to ~1% of the bunch length, which in fact makes the job of emittance preservation easier, but also brings new demands for beam diagnostics. With this difference in mind, some of the various technical challenges related to beam brightness generation, preservation, and diagnostics are described.

2 SASE FEL REQUIREMENTS

The SASE FEL requires an electron beam with ‘slice’ transverse normalized rms emittance roughly estimated by

\[ \varepsilon_{\text{nx}} \lesssim \gamma \frac{\lambda_u}{4\pi}, \]

where \( \gamma \) is the electron energy, \( E \), in units of rest mass (\( \gamma = E/mc^2 \)), and \( \lambda_u \) is the radiation wavelength (~1 Å). For \( \gamma = 3 \times 10^4 \), the emittance requirement is \( \varepsilon_{\text{nx}} \lesssim 1 \mu m \) (\( \varepsilon_{\text{x}} = \varepsilon_{\text{y}} \)). This is a challenging level for electron sources at ~1-nC, but can be eased a bit by using longer undulators. At the same time, the ‘slice’ rms \( \varepsilon^\prime \) relative energy spread should be

\[ \sigma_E/E < \rho \approx 1 \frac{1}{4} \frac{I_{\text{pk}}}{I_A} \frac{\lambda_u^2}{\beta \varepsilon_{\text{x}}} \left( \frac{K}{\gamma} \right)^{\frac{1}{3}}, \]

where \( \rho \) is the FEL parameter, \( \lambda_u \) the undulator period, \( K \) (assumed >> 1 here) the ‘planar’ undulator parameter (\( K = eB\lambda_u/2\pi mc \)), \( B \) peak undulator field, \( I_{\text{pk}} \) peak \( e^- \) current, \( I_A = 17 \) kA, and \( \beta \) the mean beta function in the undulator. For \( K = 4, \gamma = 3 \times 10^4, I = 4 \) kA, \( \beta = 20 \) m, \( \lambda_u = 3 \) cm, then \( \sigma_E/E < 0.05% \) at \( \varepsilon_{\text{x}} = 1.5 \mu m \).

The gain-length of the FEL, typically underestimated in this 1D-model, is given by

\[ L_g \approx \frac{\lambda_u}{4\pi \sqrt{3} \rho}. \]

For SASE saturation, the undulator length must be \( L_g > 20L_{\text{eq}} \), so \( L_g \) must be minimized and therefore \( I_{\text{pk}} \) maximized while preserving the transverse emittance. After accommodating some reasonable emittance growth, various undulator imperfections, and limited machine stability, plus a 3D-model, the needed undulator length can quickly stretch well beyond 100 m. The simultaneous requirement of high peak current, small energy spread, and small transverse emittance presents a significant challenge for the accelerator design.

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3 INJECTOR

The injector is typically based on an rf photocathode gun which rapidly accelerates the photo-electrons from the cathode in order to minimize the effects of space-charge forces on beam brightness. A solenoid magnet immediately after the cathode is used to focus the beam into the next accelerating section and accomplishes a compensation of the space-charge induced correlated emittance growth [11], [12]. The challenge is to extract ~1-nC charge in a ~100-A bunch with transverse normalized emittance ~1 µm. These levels have not yet been simultaneously achieved to date, but most measurements reflect the ‘projected’ emittance. Some measurements have indicated sub-micron slice emittance levels [13], [14], but at a reduced charge of 0.1-0.3 nC.

An alternate approach to high-brightness electron beam generation is being pursued in Japan using a low emittance HV-pulsed gun with a CeB₆-cathode [7] and a reduced bunch charge level of 0.1 to 0.5 nC.

In addition to generating the high brightness electron beam, the injector must serve as a stable base to operate the FEL. The cathode-illumination laser must be stable in timing, with respect to linac RF phase, to sub-picosecond levels, and laser power at the cathode (the electron bunch charge) must be stable to typically a few percent in the UV. These levels need to be achieved over time scales of a few seconds. Longer time scales will be accommodated by including timing and charge feedback systems.

4 LINAC

The linac accelerates and compresses the electron bunch, while preserving beam brightness. Acceleration reduces the geometric emittance, more closely satisfying Eq. (1), while compression increases the peak current, fulfilling Eq. (2). Bunch compression is typically accomplished by accelerating at an off-crest rf phase, providing a nearly linear energy ‘chirp’ (correlation) along the bunch length. A series of dipole magnets (usually a simple 4-dipole chicane) is used to generate an energy dependent path length so that the chirped bunch compresses in length.

One of the most challenging issues associated with magnetic bunch compression is the effect of coherent synchrotron radiation (CSR) in the bends. An electron bunch following a curved trajectory in a bend magnet will radiate energy. For wavelengths shorter than the bunch length this radiation will be coherent and at power levels greater, by the number of electrons in the bunch (N), than the incoherent component. Figure 1 shows the radiation spectrum for the final bends of the LCLS 1st-stage (BC1) and 2nd-stage (BC2) compressors. The solid curves show the coherent radiation increases by N = 6x10⁹ over the incoherent radiation (dashed), at wavelengths longer than the final rms bunch length (200 µm for BC1 and 20 µm for BC2). The incoherent power drops off as λ⁻¹/³.

The radiation field from the back of the bunch may catch up to the head of the bunch by propagating along the chord of the trajectory. The radiation field may be strong enough to alter the energy of leading electrons causing a trajectory change through the bends. The different trajectory distortions for different sections of the bunch length becomes a projected emittance growth in the bend plane, but not necessarily a slice emittance growth. Slice emittance growth, however, can also be generated by the incoherent radiation at high energy, or by the transverse gradient of the CSR longitudinal wakefield across a bunch with significant transverse extent [15].

![Figure 1: CSR power spectrum for LCLS BC1 (magenta & blue) and BC2 chicanes (green & red). Dashed curves (blue & red) show only the incoherent radiation (~λ⁻¹/³).](image)

The compressors must be designed with these detrimental effects in mind. Various design considerations have been proposed to mitigate the emittance growth [16], [17], [18].

In addition to the projected emittance growth, tracking studies [19] have also revealed a potentially more damaging CSR instability which can develop for very cold beams. Small density modulations on the current or energy profiles can be magnified by the CSR longitudinal wakefield, depending on the wavelength of the modulation, the beam’s incoherent energy spread, and the slice emittance in the bend-plane [20], [21], [22].

![Figure 2: CSR microbunching gain, Gₛ, versus modulation wavelength, λᵣ, at entrance to LCLS BC2 chicane for various ‘slice’ emittance and ‘slice’ energy spread values.](image)

Figure 2 shows gain versus modulation wavelength at the LCLS BC2 chicane entrance calculated with tracking and in theory from ref. [20]. Particle tracking of the LCLS
shows large amplification through the series of four bend systems, two of which are bunch compressor chicanes. The LCLS layout is shown schematically in Figure 3.

The microbunching effect is even more severe for the double-chicane compressor initially proposed for the LCLS BC2 [16], forcing a design change to a single-chicane. The single-chicane produces more projected emittance growth, but less microbunching, the latter being a more significant effect for the FEL.

The strong microbunching at high frequencies can increase the 'slice' emittance and the 'slice' energy spread. The effect at 14.3 GeV, at the end of the LCLS, is shown on the top 3-plots of Figure 4. The microbunching is strongly damped by including a short, one-period superconducting wiggler magnet prior to BC2 at 4.5 GeV (see bottom 3-plots of Figure 4). The wiggler increases the rms incoherent energy spread from $3 \times 10^{-6}$ to $3 \times 10^{-5}$ at 4.5 GeV, which generates a slippage across the chicane effectively smearing out the microbunching. The same damping can be achieved by increasing the bend plane slice emittance, but such freedom is not available in most emittance dominated SASE x-ray FEL's, since Eq. (1) is already violated. On the other hand, Eq. (2) is usually well satisfied, so there is head-room to add energy spread without changing the FEL gain.

Figure 5 shows simulated horizontal, $x$, versus longitudinal, $z$, position after CSR effects in LCLS with damping wiggler on. The projected emittance growth is seen as simple 'steering' of the bunch head (left) and tail.

The large compression factor required (~50) also amplifies non-linear effects due to the sinusoidal shape of the rf, longitudinal wakefields of the rf structures, and the compressor’s 2nd-order path length dependence on energy. A wakefield-induced example is clearly evident in the sharp 'horns' of the temporal distributions in Figure 4 (right-side plots). These sharp current spikes can further amplify CSR effects.

In some cases it is possible to compensate the non-linearities by including a higher harmonic rf accelerating section. This is such a natural solution that it has been independently proposed for the first compressor in TESLA [23] and the LCLS projects [24], and also for the TTF-FEL, and at Boeing [25]. In TESLA, a 3rd harmonic of L-band is used ($\beta_f = 3.9$ GHz), while LCLS uses an existing NLC X-band structure, which is a 4th harmonic of S-band ($\beta_f = 11.4$ GHz). By operating at or near the decelerating crest phase of the harmonic section with ~20 MV, both the 2nd-order curvature of the RF and the 2nd-order compression effects can be completely compensated, eliminating spikes in the compressed current distribution after the first compressor stage (BC1). This is much more difficult to compensate in the LCLS BC2, where strong longitudinal wakefields of the S-band structures and a non-uniform current profile generate a 3rd-order effect.

The differences in the TESLA and LCLS accelerator designs are most striking with regard to the accelerating RF. LCLS uses the existing SLAC linac with its S-band rf, while TESLA employs superconducting L-band rf. The accelerating gradients are similar at 18-25 MV/m, but the wakefield strengths are very different, introducing some advantages and disadvantages for each design.

Certainly the greatly reduced transverse wakefield of the TESLA design is a significant advantage, as is the lack of a strong longitudinal wakefield, which can otherwise result in the LCLS current spikes shown in Figure 4. But there is also a more subtle advantage to the strong longitudinal wakefield in the LCLS. In both machines the beam in BC2 is under-compressed, leaving a large time-correlated energy spread along the bunch. The strong longitudinal S-band wakefield can be used to completely cancel this linearly correlated energy spread prior to the undulator entrance (see center plots of Figure 4). For
To accommodate SASE saturation at ~1 Å, the SASE x-ray FEL undulator is typically quite long (120 m for the LCLS, and up to 320 m for the TESLA FEL). The trajectory requirements for such an undulator are very demanding. An imperfect trajectory will generate electron/x-ray phase slippage and loss of spatial overlap, both of which reduce the gain.

The undulator needs focusing to keep the beam size nearly constant. Quadrupole magnets inserted between undulator sections are typically used for this purpose. The quadrupoles must then be aligned very precisely or the trajectory will not be straight enough. Beam position monitors (BPMs) and steering can be used to correct the effects of misaligned quadrupoles, but the BPMs must be well aligned. For radiation wavelength \( \lambda_s (= 1.5 \text{ Å}) \), quadrupole spacing \( L \) (= 3.5 m), and number of undulator-sections \( n (= 33) \), the BPM alignment requirements (net phase slip < \( \pi \)) are estimated by [28]

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\langle \Delta x^2 \rangle^{1/2} \leq \frac{\lambda_s L}{2n}.
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For the LCLS, this requirement is \( \langle \Delta x^2 \rangle^{1/2} < 3 \mu \text{m} \). Such a level is not achievable using survey methods, so a beam-based procedure is applied to align the quadrupoles and the BPMs [29], [30]. Figure 6 shows the results of simulation of beam-based alignment for the LCLS. The final trajectory rms is 2-3 \( \mu \)m achieved using 1-\( \mu \)m resolution BPMs and scanning the beam energy over a wide range (5-14 GeV). The techniques are very reliant on BPM resolution and stability. BPMs must be built to accommodate 1-\( \mu \)m rms resolution and their readback offsets must not drift by more than 1-2 \( \mu \)m over the 1-2 hour period required to perform the procedure. Drifts due to thermal variations need to be well controlled.

Longitudinal wakefields are also an issue in the undulator. An initial correlated energy spread at undulator entrance simply generates a frequency chirped x-ray beam. But energy spread generated within the undulator of order of the FEL parameter \( \rho \) changes the FEL resonance condition during the exponential gain regime and can have a significant impact on the x-ray output power and pulse shape.

The most significant mechanism is the resistive-wall (RW) wakefield [31], which shifts the mean energy of the various slices differently, increasing the projected energy spread. For a gaussian bunch with \( N (= 6 \times 10^9) \) electrons in a cylindrically symmetric pipe of radius \( a \), the rms relative energy spread is increased by

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where \( Z_0 = 377 \Omega \), \( \sigma \) is the rms conductive, and \( \sigma_x \) is the rms bunch length. For \( a = 2.5 \text{ mm} \), \( \sigma_x = 20 \mu \text{m} \), \( E = 15 \text{ GeV} \), \( L_u = 120 \text{ m} \), and a copper pipe (\( \sigma = 5.9 \times 10^7 \text{ \Omega}^{-1} \text{m}^{-1} \)), the energy spread is 0.06% (= \( \rho \)). A larger radius helps, but it becomes difficult to produce a strong enough undulator field (\( K \)), so the undulator length must be increased, further increasing the wake. This effect limits the final bunch length, which otherwise might be further compressed to overcome emittance limitations in the gun.

5 UNDULATOR

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The RW wakefield must be evaluated over the ‘real’ non-gaussian temporal bunch distribution and the evolving energy spread included in the SASE FEL gain calculations. This has been included in the computer code Genesis 1.3 [32]. For the LCLS, the power reduction due to undulator wakefields is ~35%. Figure 7 shows the LCLS FEL output power versus distance along the undulator both with and without wakefields for a 1-nC and a 0.2-nC charge, and for 1.5-Å and 15-Å radiation. In addition to the RW wakefield, a vacuum chamber wall surface roughness wakefield can arise. Calculations for typical surfaces show this effect to be small [33].

Figure 7: LCLS FEL output power vs. distance along the undulator with (solid) and without (dash) wakefields [32].

6 REFERENCES