DIAMOND STORAGE RING APERTURES

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Abstract
This paper discusses the factors contributing to the choice of beam stay-clear (BSC) in DIAMOND. The lifetime which results from this definition is then calculated to ensure that it is adequate for the operation of the machine. The BSC is defined using a semi-empirical approach by calculating the requirement for a momentum acceptance of at least 4% at all locations around the lattice. Allowance for injection and closed orbit errors also contribute to the overall BSC.

1 FACTORS DETERMINING APERTURES

1.1 Momentum Acceptance
One of the primary factors determining the BSC is the requirement for sufficient momentum acceptance to provide an adequate Touschek lifetime. A uniform momentum acceptance of 4% is set as a target value to give an adequate lifetime in both standard operating modes – 2/3 fill multibunch (300 mA) and single bunch (10 mA), assuming a core beam coupling of 1%.

To determine the aperture requirement for each element in the lattice an acceptance of 4% needs is defined at all locations. The momentum acceptance in the horizontal plane of a particle scattered from a location \( i \) to another location \( k \) is given by:

\[
\eta = \frac{a}{H^2 \beta + D_{ik}}
\]

where \( H \) is the chromatic invariant and \( a \) is the aperture. This can be used to determine the aperture requirement at all locations, \( k \).

The equivalent expression in the vertical plane is:

\[
\eta = \frac{a}{H^2 \beta + D_{ik}}
\]

where \( H^2_{ij} = H_{ij} + kH_{ij} \). The value of \( \kappa \) - the coupling experienced by large amplitude particles - is difficult to predict [1], so the value which can be tolerated is used given the already specified vertical apertures in the insertion device (ID) straights. The variation of optical functions with momentum in equations (1) and (2) is taken account of in our calculations.

1.2 Injection
The issues of septum position, injected beam emittance, stored beam emittance and momentum error have been dealt with when determining the aperture requirements for injection. The treatment by Tazzari [2] gives a linear estimate for the optimal horizontal \( \beta \)-function for matching of the injected beam. To account for possible non-linearities we overestimate the required \( \beta \)-function by a factor of 2. Additionally:

- 5mm is allowed for the septum plate thickness, including alignment tolerances.
- Injected beam emittance up to 150 nmrad is allowed for. \( \pm 3\sigma \) is allowed either side of the injected beam, corresponding to 99.6% of the beam being captured.
- Operating modes in the storage ring with emittances up to 20 nmrad are allowed for. \( \pm 5\sigma \) is allowed either side of the stored beam.
- The storage ring is assumed to accept particles with up to 1% energy deviation.

1.3 Coulomb Scattering
The vertical apertures, determined from momentum acceptance requirements, must give an adequate Coulomb scattering lifetime. To ensure that the vertical apertures in the IDs determine the Coulomb lifetime, these apertures are scaled with the square root of the \( \beta \)-function around the ring. If the aperture requirement from this exceeds either the momentum acceptance or injection requirement at any location, the larger aperture is used.

1.4 Closed Orbit Errors
An allowance for closed orbit errors is added to the BSC. Using the assumed element errors in Table 1, a 95% confidence limit on the likely closed orbit at any location in the lattice was estimated using 300 randomly chosen error sets. The following assumptions were made:

- The likely maximum closed orbit varies around the ring. The maximum of these values is used in each plane, except in the ID straights, where no closed orbit allowance is added because the aperture is already specified to meet ID output
- No allowance has been made for an uncorrected beam to circulate in the ring. It is assumed that correction will be performed by beam threading using 1st-turn electron BPMs.

Table 1: Error table for misalignments of the storage ring.

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole Transverse Displacement</td>
<td>0.1mm</td>
</tr>
<tr>
<td>Sextupole Transverse Displacement</td>
<td>0.1mm</td>
</tr>
<tr>
<td>Dipole Transverse Displacement</td>
<td>0.05mm</td>
</tr>
<tr>
<td>Dipole Longitudinal Displacement</td>
<td>0.05mm</td>
</tr>
<tr>
<td>Dipole Field Error</td>
<td>0.1%</td>
</tr>
<tr>
<td>Quadrupole Roll Error</td>
<td>0.2mrad</td>
</tr>
<tr>
<td>Dipole Roll Error</td>
<td>0.2mrad</td>
</tr>
<tr>
<td>EBPM Resolution Error</td>
<td>1 micron</td>
</tr>
</tbody>
</table>
Although the BSC is not defined to allow an uncorrected beam to circulate, it is nevertheless likely that such a beam will be able to do so; the uncorrected orbit is not likely to exceed 34.3 mm in the horizontal plane and 17.3 mm in the vertical plane.

2 APERTURE REQUIREMENTS

2.1 Contingency

An important part in defining the BSC is the allocation of some contingency to allow for the variation of working point and optics. Using a representative sample of 15 working points (ranging over a large working region between tunes of \{22,32\} horizontally and \{10,13\} vertically), the maximum aperture that could be required in each element has been estimated; it is found that there are some elements whose maximum aperture requirements are larger than would be realistically possible, given the large demanded momentum acceptance and apertures. Some limited compromise on momentum acceptance would therefore have to be made for certain working points (although not for the reference working point).

2.2 Results

The aperture requirement for momentum acceptance for the reference working point in both planes is shown in Figure 1 and Figure 2 both for on-momentum particles and for particles with momentum deviations of ±4%. The overall BSC values are also shown. The aperture requirements for injection are shown in Figure 3 with the horizontal BSC. It can be seen that the defined apertures are also large enough for both requirements.

Figure 1: Horizontal aperture requirements for momentum acceptance. The BSC is shown in green, whilst the overlapping red, blue and black curves show the required aperture for 4% momentum acceptance calculated using optical functions at –4%, 0 and +4% momentum deviation respectively.

Figure 2: Vertical aperture requirements for momentum acceptance. The BSC is shown in green, whilst the overlapping red, blue and black curves show the required aperture for 4% momentum acceptance calculated using optical functions at –4%, 0 and +4% momentum deviation respectively.

Figure 3: Aperture requirements for injection. The BSC is shown in green, whilst the red, blue and black curves show the required horizontal aperture for injected particles with –1%, 0% and 1% energy mismatch.

3 LIFETIME

3.1 Touschek Lifetime

The Touschek lifetime resulting from the definition of the BSC has been calculated using Brück’s method, which is a sufficiently good approximation given other uncertainties [3]. The lifetime is given by

\[
\frac{1}{\tau} = \left[ \frac{\pi^{1/2} c N_a C(\varepsilon)}{\gamma^2 V \eta^2} \right] \left( \delta q \right)_\text{mod}^2,
\]

where \( V \) is the bunch volume, \( \varepsilon = (\eta/\delta q)^2 \) is a scaled measure of the ‘space’ available to the beam, and \( C(\varepsilon) \) a loss rate function given by:

\[
C(\varepsilon) = -\frac{3}{2} e^{-\varepsilon} + \frac{\varepsilon}{2} \int_0^\infty \ln u e^{-u} du - \frac{3\varepsilon - \varepsilon^2}{2} \int_0^\infty \frac{e^{-u}}{u} du.
\]

\( \delta q \) is the rms transverse momentum given by:

\[
\delta q = \gamma \sigma_{q,\text{mod}} = \frac{\sigma_y}{\sigma_z} \sqrt{1 + \frac{H\sigma_y^2}{\varepsilon_z}}.
\]
A scattering rate is calculated at each location in the lattice, which is then averaged by scaling with the length of each element to give the overall Touschek lifetime. The momentum acceptance used is the minimum of the acceptance from the physical apertures, the acceptance from the dynamic aperture and the RF acceptance, which is specified to be 4% (constant around the storage ring).

The momentum acceptance from the dynamic aperture was calculated by tracking a set of particles with a range of momentum deviations scattered from all locations in the lattice. The largest momentum deviation which survived was taken to be the dynamic momentum acceptance at that point; these results are shown in Figure 4. Positively and negatively scattered particles can have different dynamic acceptances and so these are estimated separately.

![Figure 4: Dynamic momentum acceptances for positively and negatively scattered particles, obtained by tracking over 1024 turns.](image)

Since the likely degree of bunch lengthening is difficult to predict, the natural (zero-current) bunch length of 3.26 mm has been used to calculate a lower limit on the Touschek lifetime. Assuming a beam current of 300 mA and 2/3 of the bunches filled (to avoid significant ion trapping [4,5,6]), the predicted Touschek lifetime is 75.3 hours. Assuming a single bunch beam current of 10 mA the predicted Touschek lifetime is 3.6 hours.

3.2 Gas Lifetime

There are four scattering processes the beam electrons can undergo with the residual gas in the storage ring [7]. These are Coulomb and Bremsstrahlung interactions of electrons with the gas nuclei and elastic and inelastic interactions with the gas electrons. The cross sections are:

\[
\sigma_B = \frac{16r_e^2Z^2}{411} \ln \left( \frac{183}{z^3} \right) \left( \ln \frac{1}{\eta} \right) \left( \frac{5}{8} \right)
\]

\[
\sigma_e = \frac{4\pi e^2Z^2}{2\gamma^2} \langle \beta \rangle \frac{\beta}{a_{max}^2}
\]

It is assumed that the residual gas pressure will be lnTorr, composed solely of molecular nitrogen and that its temperature will be 300K. This is a reasonable approximation to the likely gas composition, and for the pressure likely to be obtained after some conditioning of the storage ring.

It can be seen in Table 2 that, as expected, the most significant contributions to the gas lifetime are the Bremsstrahlung and Coulomb components. The inelastic component makes a small contribution while the elastic contribution is insignificant.

<table>
<thead>
<tr>
<th>Component</th>
<th>Lifetime Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bremsstrahlung</td>
<td>76.4 hours</td>
</tr>
<tr>
<td>Coulomb</td>
<td>91.6 hours</td>
</tr>
<tr>
<td>Inelastic</td>
<td>211 hours</td>
</tr>
<tr>
<td>Elastic</td>
<td>8470 hours</td>
</tr>
<tr>
<td>Total Gas</td>
<td>34.6 hours</td>
</tr>
</tbody>
</table>

3.3 Total Lifetime

By combining the contributions from Touschek lifetime and gas lifetime, we estimate the total lifetime in standard multibunch operation will exceed 23 hours; this comfortably exceeds the design specification of 10 hours. The total lifetime in single bunch mode is estimated to be over 3 hours. However, with the significant bunch lengthening that is likely to occur in this mode the lifetime will probably also exceed 10 hours. We should point out that all lifetime estimates like those presented in this paper are subject to considerable uncertainty, due to the difficulty in predicting crucial parameters such as residual pressure, bunch lengthening, beam coupling and dynamic aperture. A factor of safety in design commensurate with this uncertainty should be made in all such calculations.

4 REFERENCES