SPACE CHARGE SIMULATION IN COSY USING FAST MULTIPOLE METHOD

B. Loseth*, M. Berz*, K. Makino*, H. Zhang*, P. Snopok**
* Michigan State University, East Lansing, MI 48824, USA
** Illinois Institute of Technology, Chicago, IL 60616, USA

Abstract

A method is presented that allows the computation of space charge effects of arbitrary and large distributions of particles in an efficient and accurate way based on a variant of the Fast Multipole Method (FMM). It relies on an automatic multigrid-based decomposition of charges in near and far regions and the use of high-order differential algebra methods to obtain decompositions of far fields that lead to an error that scales with a high power of the order. Given an ensemble of N particles, the method allows the computation of the self-fields of all particles on each other with a computational expense that scales as O(N). Using remainder-enhanced DA methods, it is also possible to obtain rigorous estimates of the errors of the methods. Furthermore, the method allows the computation of all high-order multipoles of the space charge fields that are necessary for the computation of high-order transfer maps and all resulting aberrations. This method has been implemented in COSY Infinity, and the progress of applying it to simulating the 6D cooling channel for the Muon Collider is reported.

FAST MULTIPOLE METHOD

The fast multipole method for the Coulomb interaction between many charged particles divides an arbitrary charge distribution into small boxes with a hierarchical structure. It then computes the multipole expansions and local expansions of charges far from the observer to achieve a computation efficiency that scales with the number of particles, N, and computational errors scaling with a high power of the differential algebra order. The FMM algorithm is especially suited for beam dynamic simulations because of the efficiency and low computational error compared to other spacecharge algorithms.

Other methods, such as the particle-particle interaction (PPI) and the particle in cell (PIC) method, while computationally efficient, incur excess error due to modeling and statistics. The PPI method uses macroparticles and assumes a particular distribution. The PIC method places the charge distribution onto a mesh, solves the Poisson equation on mesh points and interpolates between mesh points to find the field on each particle. Both of these methods suffer from an inability to precisely handle complicated charge distributions. This difficulty is overcome in the FMM by decomposing the charge distribution into boxes according to the charge density such that there are a pre-specified number of particles in each box to efficiently and accurately compute the multipole expansions.

FMM Algorithm

The principle of the FMM is to determine the potential of groups of source particles sufficiently far from the observer in terms of expansions in $1/r$, utilizing the fact that far away, high powers of $1/r$ become less significant. These multipole expansions can be translated and combined, and again locally expanded to determine the field on a group of nearby observer particles.

First, all of the charges are enclosed in a cube box called the zero-level box. A box is defined to be a parent box if the number of charges inside is larger than s, which we select. If a box is a parent box, it will be split into eight equal square boxes which are its child boxes and form a new level. If any of the child boxes have more than s charges, they will in turn be split into eight child boxes which form the next level. This process is iterated until no box has more than s charges. At each subdivision, we keep only the nonempty boxes, so the empty boxes are ignored by the subsequent field calculation.

Analytic Considerations

For a distribution of n particles of charge q_i located at $\mathbf{r_i}(x_i, y_i, z_i)$, the potential at a point $\mathbf{r}(x, y, z)$ can be expressed as

$$\phi = \sum_{i=1}^{n} \frac{d_r \cdot q_i}{\sqrt{1 + r_i^2 d_r^2 - 2x_i d_x - 2y_i d_y - 2z_i d_z}} \quad (1)$$

with

$$d_x = \frac{x}{x^2 + y^2 + z^2} \quad d_y = \frac{y}{x^2 + y^2 + z^2}, \quad (2)$$

$$d_z = \frac{z}{x^2 + y^2 + z^2} \quad d_r = \frac{1}{\sqrt{x^2 + y^2 + z^2}}. \quad (3)$$

In COSY Infinity [1,2] we apply the DA techniques [3,4] to express ϕ as a DA vector by choosing d_x, d_y, and d_z as our DA variables. This DA vector is essentially the Taylor expansion of ϕ with respect to $1/d_x$, $1/d_y$, $1/d_z$, $1/d_r$, and can be translated into another multipole expansion at an arbitrary point and converted into a local expansion with a simple composition operation on the DA vector. Using the domain decomposition to determine near and far regions, the field in a box is computed using a combination of far multipole expansions and local expansions.

LINEAR COOLING CHANNEL IN COSY

The demonstration of muon ionization cooling is one of the key challenges for the Muon Accelerator Program

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hosted at Fermilab [5]. A linear cooling channel consists of solenoids, RF cavities, and wedge absorbers designed for the transverse cooling of a muon beam (Figure 1). Longitudinal emmitance is not reduced in this channel; slower particles lose more energy in absorbers than their faster counterparts, leading to an increase in longitudinal emittance. As the overall beam size is reduced, the density of particles is increased which leads to space charge effects becoming significant. Studying these space charge effects requires developing tools in COSY that are coherent with the FMM.

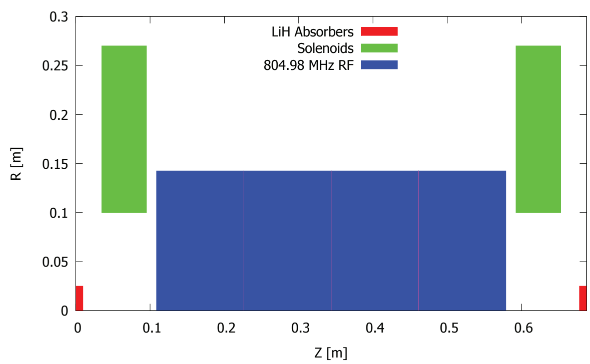

Figure 1: R vs. Z geometry of 1 cell.

Ionization Cooling

Muons are produced by sending proton bunches into a target in order to create pions, which in turn decay into muons. As a consequence of this production, the muons are quite 'hot' by having a large momentum spread. The beam must be cooled significantly, within the muon lifetime, in order ensure a high luminosity muon collider. Beam emittance is reduced via ionization cooling; the beam is strongly focused with magnetic fields and subsequently sent through an absorber material to reduce overall momentum. The beam regains longitudinal momentum in RF cavities, resulting in a net loss in perpendicular momentum and thus beam emittance [6].

Solenoids

The cooling channel is divided into 10 cells with two solenoids of opposite polarity per cell. These solenoids are placed on either side of the absorber wedges that divide the cells in order to increase transverse focusing at the absorber, which in turn minimizes the heating due to multiple scattering.

To implement these solenoids in COSY, a 2-dimensional fieldmap with sufficient range was produced using a Taylor Model based integrator. For a thick solenoid with uniform azimuthal current and finite length, the field at an arbitrary point in space $\mathbf{r}$ is given by the Biot-Savart Law

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{(\mathbf{J}dV) \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} . \quad (4)$$

To perform this integral, the domain is subdivided into small cylindrical boxes centered at $\mathbf{r}'$, and the taylor model of the integrand is computed. If the remainder bound of this taylor model is larger than a predefined error bound, the box at $\mathbf{r}'$ is subdivided into 8 equal boxes and the taylor model is computed again for the child boxes. This process is repeated until the remainder bound criterion is met, and integrated over the entire domain of current. Integrating the domain in this manner is computationally efficient because the areas of the domain that are far from the observation point are not subdivided as finely, so less time is spent on the domain compared to a uniform domain decomposition.

To generate the field on a particle in the ring:

- The particles are translated, rotated, and tilted into the first solenoid's frame of reference
- Using linear interpolation between field map points, the magnetic field on each particle is determined in each direction
- The resulting magnetic field in the solenoid's frame is then tilted and rotated back into the global frame and added to the total field
- This process is repeated for each solenoid in the channel
- For this channel, the solenoids have no rotation or tilt, so a 2D fieldmap for an entire cell was produced to take advantage of the cylindrical symmetry

RF Cavities

Each cell in the ring contains four closely spaced RF cavities which fit inside the solenoids. Similar to the solenoids, a fieldmap is used to determine the electric and magnetic fields on the particles inside the cavity. The timing of the $\omega = 804.98$ MHz cavities is determined by the time the reference particle is tracked through the center of the cavity. In order to achieve the correct RF phase (φ_0) at the center of the cavity, the tuning process proceeds as follows:

- Stop the reference particle at the boundary of the cavity and create an identical tune particle
- make a crude guess of the offset time: $t_0 = RFlength/2v$
- propagate the tune particle to the center of the cavity using the crude offset time for the RF fields
- update the offset time: $t_0' = t + 0 + dt$, where $dt = (\varphi - \varphi_0)/\omega$
- reload the tune particle at the beginning of the cavity with the updated offset time
- exit the tuning loop once dt is sufficiently small

Absorber Wedges

The absorber wedges are placed at the boundary of each celll. The energy loss of a particle over a distance ds in the absorber is determined by the Bethe-Bloch formula

$$\frac{dE}{ds} = -K\rho \frac{Z}{A} \frac{z^2}{\beta^2} (log(\frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2}) - 2\beta^2 - \delta), \quad (5)$$

for the parameters

- $K = 15.35375(MeV \cdot cm^3)/(m \cdot g)$,
- Z is the atomic number of the absorber material (LiH),
- A is the atomic mass of the absorber material,
- ρ is the density of the absorber material,
- I is the ionization potential in MeV,
- δ is the density correction parameter,

where the maximum kinetic energy transferred to a single electron in the absorber in a collision is

$$T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/m_\mu + (m_e/m_\mu)^2}. \quad (6)$$

SIMULATIONS IN COSY AND G4BEAMLINE

With all of the lattice elements previously discussed implemented in COSY, we proceeded to compare simulations of a 10-cell cooling channel in both COSY Infinity and G4beamline [7] with and without space charge effects. G4beamline is one of the de-facto codes for muon beam simulations and provides a space charge calculation algorithm and is useful to benchmark for the new COSY functionality. Stochastic effects and particle decays were turned off in both codes to ensure an accurate comparison; these processes are not yet implemented in COSY.

We start with the same beam of 1000 particles in both codes and compare the average orbit excursion, average momentum (Figure 2) and finally, transverse and longitudinal emittance (with and without spacecharge; Figures 3, 4). As mentioned previously, longitudinal cooling is not achieved in this linear channel; this region of the Bethe-Bloch function has a negative slope (ie slower particles lose more momentum). The behavior of the emittances is noticeably different between the codes although the form of the emittance graphs is similar.

Conclusions

In conclusion, we have gained a lot of insight into cooling channel simulations. We can simulate various lattice elements and compare meaningful figures between the codes with and without spacecharge. As this is a work in progress, some of the issues and differences evident during the cross-check with other codes are not resolved yet and require further investigation. There is still a lot of effort to be made on a detailed result comparison in order to obtain consistent results upon comparing the codes, as well as general improvment to efficiency and functionality in COSY.

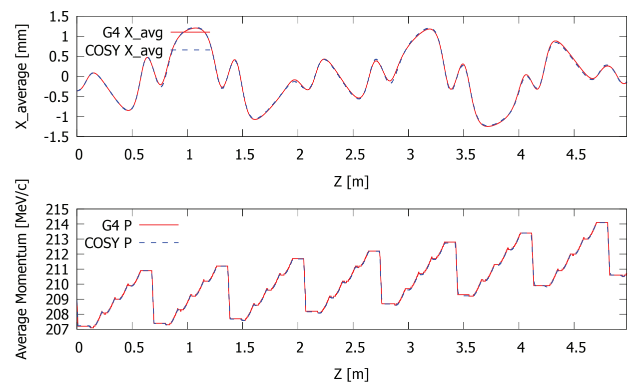

Figure 2: Average Momentum and X as a function of Z for the beam in G4beamline and COSY.

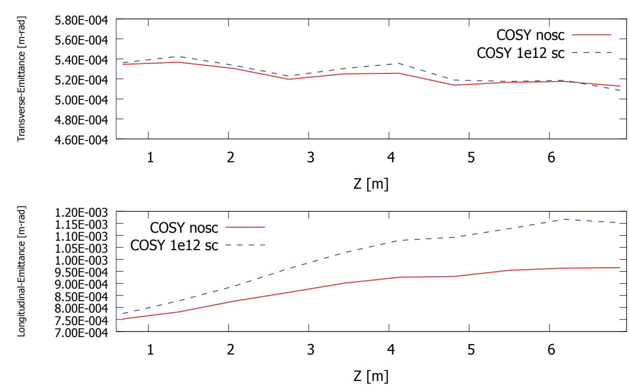

Figure 3: Transverse and Longitudinal beam emittance over 10 cells in COSY with and without space charge.

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