ADVANCED MODELING OF TE MICROWAVE DIAGNOSTICS OF ELECTRON CLOUDS

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Abstract

Numerical simulations of electron cloud build up and in particular rf microwave diagnostics provide important insights into the dynamics of particle accelerators and the potential for mitigation of destabilizing effects of electron clouds on particle beams. Typical Particle-In-Cell (PIC) simulations may accurately model cloud dynamics; however, due to the large range of temporal scales needed to model side band production due to ecloud modulation, typical PIC models may not be the best choice. We present here preliminary results for advance numerical modeling of rf electron cloud diagnostics, where we replace kinetic particles with an equivalent plasma dielectric model. This model provides significant speedup and increased numerical stability, while still providing accurate models of rf phase shifts induced by electron cloud plasmas over long time scales.

PLASMA DIELECTRIC MODELS CAPTURE ESSENTIAL PLASMA RESPONSE TO RF

Electron Cloud (EC) simulations using Particle-In-Cell Codes (PIC) provide a powerful tool for understanding cloud build up, mitigation techniques, as well as traveling TE microwave diagnostics of electron clouds. However, to explicitly model side bands induced in TE waves due to an electron cloud plasma, one must simulate beam revolution time scales (the cloud modulation time) but still resolve the rf signal. Modeling electron clouds as kinetic particles is time consuming (particle pushes are slow compared to field updates) and numerically noisy over long simulation times (grid heating). One solution is to replace kinetic particles with an equivalent plasma dielectric model. Plasma dielectric models of electron clouds are much faster, and are more stable numerically.

Figure 1 shows four time slices of a typical kinetic PIC simulation of electron cloud buildup. Cold electrons are initially seeded around the center of the beam pipe (far left). As a particle beam passes through the electron cloud the electrons are accelerated toward the (positive) potential of the beam (second from left), and drift to the beam pipe walls in the period between beam bunched. As they impact beam pipe walls, they emit secondary electrons (second from right), which are accelerated laterally across the beam pipe chamber by subsequent beam bunched (far right).

Figure 1: Time sequence of a typical PIC simulation of electron cloud buildup using kinetic particle models (PIC simulations).

For a single-species, unmagnetized plasma, the linear plasma dielectric constant is simply \( \epsilon = 1 - \frac{\omega_p^2}{\omega^2} \), where \( \omega_p^2 = 56.4 \sqrt{n_e} \) is the plasma frequency [1]. In time-domain simulations using the VORPAL plasma simulation code [2], we incorporate this dielectric by converting frequencies to time derivatives, and introducing an auxiliary vector \( J \) representing the linear currents induced in the plasma. We can then solve Ampere’s law in the time domain with an additional update equation for the linear currents,

\[
\epsilon_0 \partial_t E = -J - \frac{1}{\mu_0} \nabla \times B \quad (1)
\]

\[
\{\partial_t + \nu\} J = \epsilon_0 \omega_p^2 E - \Omega \times J \quad (2)
\]

where \( \nu \) is the collision frequency and \( \Omega \) is the electron gyrofrequency. The plasma dielectric model for TE microwave electron cloud diagnostic simulations provide significant speedup over kinetic PIC simulations while still maintaining accuracy in phase shifts induced due to the plasma. For instance, simulations performed here, with a typical grid size of 512x8x8 performed for 10,000 rf cycles (~ 800,000 time steps) takes approximately 20 minutes of wall time, or about 1.5 ms/time step. This is more than 15x faster than an equivalent kinetic particle PIC simulation with 500,000 particles.

In addition, the plasma dielectric model allows for external magnetic fields to be added in a natural way, by simply replacing the plasma dielectric constant \( \epsilon \) with a dielectric tensor. The plasma dielectric model accurately captures the frequency dependence of the plasma response to transmitted rf signals, and collisional plasmas can be modeled.
through the inclusion of the loss parameter $\nu$. Since the plasma plasma dielectric depends on the electron density in a straightforward way, it is possible to accurately model spatially non-uniform electron clouds. The CFL condition, which determines the needed time and spatial resolution for accurate simulations remains unchanged with this method. Simulations will show better scaling to more processes (when run in parallel) because the plasma dielectric simulations only require field updates (no particle pushes).

**SIMULATION RESULTS**

We have simulated the generation of side bands observed in TE microwave diagnostics due to electron cloud modulation by directly modulating the plasma dielectric constant harmonically. In these simulations we demonstrate this technique on a plane wave system with a carrier frequency of 2.5 GHz, and modulate the plasma dielectric at a frequency of 1.0 GHz. We numerically measure the rf electric field after transmission through 0.48 m of plasma, for a range of equivalent densities ranging from $1.0 \times 10^{14}$ to $1.0 \times 10^{15}/m^3$. The geometry of the plasma dielectric simulations is shown in Figure (2). These simulations are nearly 1-Dimensional, with a longitudinal (along beam) length of 0.48 m, and a transverse length of 0.06 m. The transverse direction has periodic boundary conditions, while the longitudinal direction has Matched Amplitude Layers (MALS) which absorb wave energy.

![Figure 2: Simulation geometry for plasma dielectric models of TE microwave electron cloud diagnostics.](image)

To first order, the side band amplitudes are linearly related to the induced phase shift, which has been shown in simulations and experiments to be approximately linear with average cloud density [3]. Linear theory, valid for uniform density electron clouds, predicts that the relative amplitude of side bands can be expressed in terms of the electron cloud density as (e.g. [4]),

$$n_e = \left( \frac{8\pi m_e \epsilon_0}{e^2} \right) \frac{A_s}{A_c} \frac{c}{L} \frac{\sqrt{f^2 - f_e^2}}{f^2} \quad (3)$$

where $A_c$ is the amplitude of the carrier frequency, $A_s$ is the amplitude of the first side band, $f$ is the carrier frequency, $f_c$ is the cutoff frequency, and the units are in MKS. In the plane wave case, $f_c = 0$. Figure (3[left]) shows simulated spectra as a function of equivalent electron cloud density. First order side bands are observed in these simulations. While it is difficult to experimentally back out the electron cloud density from TE traveling wave side bands, our simulations are able to directly correlate side band amplitudes with electron cloud densities.

![Figure 3: Simulated spectrum derived from direct harmonic modulation of plasma dielectric constant in simulations (left). First side band amplitudes as a function of cloud density for three different spatial distributions. In all simulations the overall cloud density measured over the entire space is equal (right).](image)

We also measure the effect of non-uniformity in the cloud by simulating additional cases where the cloud is confined to smaller volumes ($L_x \times L_y/2$ and $L_x \times L_y/4$) while the total number of particles is kept constant. One might expect that this method just increases the equivalent cloud density. However, the effect of having the rf wave travel partially in the plasma, and partially in vacuum is not to simply average the dielectric constant. Fig. (3[right]) compares the simulated side band heights as a function of equivalent electron cloud density. As can be seen in the figure, in each case the side band amplitude ratio is linear in equivalent cloud density, as expected from Eq. (3). However, the slope with respect to cloud density increases with more non-uniformity. This indicates that it is the portion of the cloud that is sampled by the rf that contributes to the side bands, not the overall density. In addition, since the transverse direction is periodic in these simulations, the plane wave is actually sampling a semi-infinite grid of dielectric/vacuum interfaces. The cause of the differences in slope is currently under investigation.

**REFERENCES**


