USING AN EMITTANCE EXCHANGER AS A BUNCH COMPRESSOR*


Abstract

An Emittance EXchanger (EEX), like a chicane, can be used for bunch compression. However, it offers a unique characteristic: the RS6 term in an EEX vanishes, which decouples the final longitudinal position from the particles’ energies, thereby suppressing the microbunch instability and providing a great deal of flexibility in tailoring the final particle longitudinal phase space.

DEFINITION OF AN EEX

The Emittance EXchanger (EEX) is a remarkable example of the conservation of eigen-emittances [1]. Through it, a beam’s longitudinal emittance is swapped with one of its transverse emittances (which, from this point on, we assume is the horizontal dimension). The EEX process was first proposed in [2] by inserting a transversely deflecting rf cavity in the middle of a chicane, to generate transverse-longitudinal coupling. In 2006, it was realized that the exchange can be made exact (in a linear sense) if the transversely deflecting rf cavity is between two doglegs with the same orientation, for a thin rf cavity [3,4]. The EEX was first experimentally demonstrated in 2009 with that configuration [5]. It was recognized in [4] that second-order dispersion can lead to growth of the final horizontal emittance, and that can be minimized by employing an initial energy chirp to the beam (which is also used to minimize the effect of the thickness of the rf cavity).

The basic EEX optics are shown in Fig. 1. An EEX consists of two doglegs separated by a transversely deflecting rf cavity, of normalized amplitude \( k = 2\pi V_{\text{deflecting}} / \lambda E \), where \( E \) is the beam energy. If the deflecting cavity strength is chosen to be \( k\eta = -1 \), where \( \eta \) is the dispersion of the doglegs, the net linear transfer matrix for a particle vector of the form \( \mathbf{x} = (x, x', z, \Delta y / \gamma) \) is (assuming a thin rf cavity):

\[
M_{\text{dog}}M_{\text{kicker}}M_{\text{dog}} = \begin{pmatrix}
0 & 0 & -L & \frac{\eta - \varepsilon}{\eta} \\
0 & 0 & -1 & -\frac{\varepsilon}{\eta} \\
\frac{\varepsilon}{\eta} & \frac{\eta - \varepsilon}{\eta} & 0 & 0 \\
-\frac{1}{\eta} & \frac{L}{\eta} & 0 & 0
\end{pmatrix}
\]

where \( \varepsilon \) is the doglegs’ time dispersion and \( L \) is the effective length of each dogleg (including the drifts to the rf cavity). In terms of the dogleg parameters, the length, dispersion, and time dispersion are:

\[
L = S_1 \frac{1}{\cos^3 \theta_0} + 2 \frac{D}{\cos \theta_0} + S_2
\]

\[
\eta = S_1 \sin \theta_0 \cos^2 \theta_0 + 2 \frac{D}{\sin \theta_0} \left( \frac{1}{\cos \theta_0} - 1 \right)
\]

\[
\varepsilon = S_1 \sin^2 \theta_0 \cos^3 \theta_0 + 2 \frac{D}{\sin \theta_0} \left( \frac{\sin \theta_0}{\cos \theta_0} - \theta_0 \right)
\]

It should be noted that a series of thick rf cavities can be manipulated into effectively acting as a single thin rf cavity, by separating groups of rf cavities with optics that are effectively negative horizontal drifts [6].

![Figure 1: Basic EEX optics.](image)

A transfer matrix of the form shown in Eqn. (1) will swap the horizontal and longitudinal emittances. Since the form of Eqn. (1) also shows that the final longitudinal beam length only depends on the initial horizontal beam parameters, an EEX can be designed to compress an electron bunch with significantly different characteristics as seen with compression in a chicane. Note that if \( \eta - \varepsilon L / \eta = 0 \) the final particle longitudinal position only depends on the initial particle horizontal position, which provides an extraordinary ability to control the final longitudinal distribution. That constraint can only be met if \( L < 0 \), which implies that the drift \( S_2 \) includes optics that make it effectively negative for the horizontal direction.

ALTERNATIVE EEX CONFIGURATION

If we consider adding additional optics to the base EEX design we can obtain significantly more control in compressing the bunch, for an initially uncorrelated beam. (Alternatively, these optics can be thought as initial horizontal beam correlations. In either case, we assume the beam’s initial longitudinal phase space is uncorrelated.) If we precede the EEX with a drift of length \( a \) followed by a lens of inverse focal length \( -b \), the final particle vector in terms of an initial particle vector is given by

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For this case, the more generalized constraint that makes the final position only depend on initial horizontal position is \((\eta - cL/\eta)(1 + ab) = a\eta / \eta\). The final bunch length in terms of the initial horizontal size is then

\[
\frac{L}{\eta} z_i + \left(\eta - \frac{cL}{\eta}\right) (\Delta \gamma / \gamma) i
\]

and the final transverse size is given by

\[
x_{f,\text{rms}} = \frac{\frac{c}{(1 + ab)\eta} \left[ x_i^2 \right]^{1/2} + \frac{\eta a}{L + Lab + a} \left( (\Delta \gamma / \gamma) \right)^2}{\left[ \frac{L}{\eta} \left( \left< z_i^2 \right> + \frac{\eta a}{L + Lab + a} \left( (\Delta \gamma / \gamma) \right)^2 \right) \right]^{1/2}},
\]

(7)

There are unlimited design possibilities using these parameters. Two compelling choices are picking both \(b = 0\) and \(L = 0\), which we call case C, and picking \(a = L - cL^2 / \eta^2\) and \(bL = -1\), which we call case G [6]. Case C is compelling because it minimizes the final horizontal beam size (and thus the emittance growth from second order effects) and Case G is compelling because it decouples the final longitudinal phase space, leading to the smallest possible final energy spread (allowing for compression at the highest possible energy). The final longitudinal particle parameters for case G are \(z_f = b \eta x_i\) and \((\Delta \gamma / \gamma)_f = x_i^\prime / b\eta\).

**EEX COMPRESSION PERFORMANCE**

At this point we compare EEX cases C and G to compression in a chicane, all for a 1-GeV electron bunch, using the 1-D CSR model in PUSHER. We consider nominal EEX parameters where \(D = S_1 = 10\) m, with 3.5 degree bends, which leads to \(\eta = -1.225\) m and \(\varepsilon = 0.06241\) m, and where \(S_2\) is picked to satisfy the horizontal to axial mapping constraint. The nominal horizontal to longitudinal compression factor for case C, \(\varepsilon / \eta\), is 0.05095 (about a factor of 20). To end up with a 25-fsec long bunch (7.5 microns), we start with a beam with an rms horizontal size of about 150 \(\mu\)m. To have similar compression for case G we pick \(a = -1.0954\) m, \(b = -0.0399\) m, and \(S_2 = 5\) m, which leads to a horizontal-to-axial compression of 0.0488. For a comparative chicane, we also assume 10-m dipoles and drifts, and have a nominal final bunch length of 25.7 fsec for an rms energy slew of 1.14 MeV (about a 0.1% energy slew). In the following simulations, we vary the bunch charge up to 1 \(nC\), which would result in a ridiculously high compressed current of up to 40 kA. It should be noted that the purpose of these simulations is to provide a comparison between the types of effects CSR introduces in a chicane and an EEX. For all these simulations, we assume an initial normalized horizontal and an initial normalized longitudinal emittance of both 1 \(\mu\)m.

In Fig. 2(a), we compare the horizontal and longitudinal emittance for the EEX cases (a) and the final longitudinal phase spaces, for very low bunch charges, (b) Case C, (c) Case G, and (d) chicane. The initial longitudinal phase space was numerically populated with a Gaussian distribution in both position and energy deviation whereas the initial transverse phase spaces (and thus the final longitudinal phase space) were populated with a 4D K-V distribution.
emittance growth in the chicane is clearly visible as the longitudinal energy spread distorts from the CSR wake, while it is suppressed by the weaker energy-phase correlation in the EEX cases. The final longitudinal phase space plots for Case G (Fig. 5) show the least structure from CSR, resulting in the lower axial emittance plotted for that case in Fig. 2(a) and the reduced sensitivity to final bunch length plotted in Fig. 2(b). The horizontal emittance growth in the EEX cases is due to the x-z coupling induced in the exchange.

Next, we qualitatively consider susceptibility to the microbunch instability. Numerically, we axially modulate the initial beam distribution with a 25% harmonic current at a wavelength of about 10 μm. The final longitudinal phase spaces for EEX Case C and the chicane are seen in Fig. 14, both for bunch charges of 200 pC. The instability is present for both cases, but is more significant in the chicane compressor, as expected. The initial modulation leads to an enhanced energy banding in the EEX compressor, but there is no residual coupling to the final axial position (which indicates instability suppression). The emittance growth from the microbunch features, largely longitudinal for a chicane, is only horizontal for the EEX. The emittance grows from $\varepsilon_x/\varepsilon_z = 9.6/5.3$ μm without seeding to 16.3/5.3 μm with seeding for EEX Case C and from 7.7/12.0 μm to 7.7/18.6 μm for the chicane case.

Figure 4: (a) Final longitudinal phase for a 100 pC bunch compressed in EEX Case C and (b) final longitudinal phase for a 1 nC bunch compressed in EEX Case C.

Figure 5: (a) Final longitudinal phase for a 100 pC bunch compressed in EEX Case G and (b) final longitudinal phase for a 1 nC bunch compressed in EEX Case G.

Figure 6: (a) Final longitudinal phase for a 100 pC bunch compressed in the chicane and (b) final longitudinal phase for a 1 nC bunch compressed in the chicane.

Figure 7: Final longitudinal phase spaces showing the microbunch instability for compression in a (a) chicane and in (b) EEX Case C, all for 200 pC.

REFERENCES

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