SPIN-MANIPULATING POLARIZED DEUTERONS*

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Abstract
Spin dynamics of polarized deuteron beams near depolarization resonances, including a new polarization
preservation concept based on specially-designed multiple resonance crossings, has been tested in a series of
experiments in the COSY synchrotron. Intricate spin dynamics with sophisticated pre-programmed patterns as
well as effects of multiple crossings of a resonance were studied both theoretically and experimentally with
excellent agreement. Possible applications of these results to preserve, manipulate and spin-flip polarized beams in
synchrotrons and storage rings are discussed.

INTRODUCTION
Polarized hadron and lepton beams are used to study the spin dependence of hadronic interactions in the multi-
GeV/c region. Precise polarized scattering experiments require frequent spin-direction reversals to reduce
systematic errors. Moreover, one must efficiently overcome spin resonances to maintain the polarization.

The spin state of a spin-1 particle beam is described by vector and tensor polarizations [1]:

\[ P_v = \frac{N_+ - N_-}{N}, \quad P_T = 1 - 3 \left( \frac{N_0}{N} \right), \]

where \( N_+ \), \( N_0 \), and \( N_- \) are the number of particles in \( m = +1 \), \( 0 \), and \( -1 \) states and \( N = N_+ + N_0 + N_- \) is the total number of particles.

In flat circular rings, each deuteron’s spin precesses around the vertical fields of the ring’s dipole magnets, except near a spin resonance. The spin tune \( \nu_s \) (the number of spin precessions during one turn around the
ring) is proportional to the deuteron’s energy \( \nu_s = G \gamma \), where \( G \equiv (g - 2)/2 = -0.142987 \) is the deuteron’s gyromagnetic anomaly and \( \gamma \) is its Lorentz energy factor.

The deuteron’s polarization can be perturbed by the horizontal rf magnetic field from either an rf solenoid or
an rf dipole. At a resonant frequency the perturbations can add coherently to induce an rf spin resonance. The rf-
induced spin resonance’s frequency is

\[ f_r = f_c (k \nu_s), \]

where \( f_c \) is the deuterons’ circulation frequency and \( k \) is an integer. A stored beam’s polarization can be
manipulated in a well-controlled way by ramping an rf magnet’s frequency through an rf-induced spin resonance [2-10].

EXPERIMENTAL RESULTS
The apparatus used in our experiments included the COSY storage ring, the EDDA detector, the low energy
polarimeter, the injector cyclotron, the polarized ion source, and the rf dipole or rf solenoid [4-10]. The beam
from the polarized \( D^- \) ion source was accelerated by the cyclotron to 75.7 MeV and then strip-injected into COSY.
When needed, the beam was electron-cooled in COSY for up to 25 s at the injection energy. The deuterons were
then accelerated to 1.85 GeV/c, where their average circulation frequency \( f_c \) was 1.14743 MHz and their
Lorentz energy factor was \( \gamma = 1.4046 \). With these parameters, the spin tune \( \nu_s = G \gamma \) was \( 0.020884 \).

The rf dipole consisted of an 8-turn ferrite-core water-cooled copper coil with the spacing between its turns
optimized to produce a uniform radial magnetic field; it ran as a part of an LC resonant circuit giving an \( \int B \cdot dl \) of

\[ 0.54 \pm 0.03 \text{T-mm rms at 917 kHz. It was later replaced with a 25-turn air-core water-cooled rf solenoid, which
produced an rf \( \int B \cdot dl \) of 0.67 \pm 0.03 T-mm rms at the same frequency.} \]

The EDDA polarimeter measured the beam’s polarization in COSY. We reduced its systematic errors by
repeatedly cycling the polarized deuterion ion source beam through five spin states with nominal vector \( P_v \) and
tensor \( P_T \) vertical polarizations:

\( (P_v, P_T) = (0,0), (+1,+1), (-1/3,-1), (-2/3,0), (-1,+1). \)

The measured (0,0) state polarization was subtracted from each of the other measured polarizations to correct
for detector efficiencies and beam motion asymmetries.

Spin Flipping
Ramping an rf magnet’s frequency through the spin resonance frequency \( f_c \) can rotate the deuteron beam’s polarization. The modified [2] Froissart-Stora formula [11] gives the beam’s final vector \( P_v \) and tensor \( P_T \) polarizations [3] after such a ramp in terms of the beam’s initial vector \( P'_v \) and tensor \( P'_T \) polarizations, the spin resonance strength \( E \), the ramp’s frequency range \( \Delta f \) and its ramp time \( \Delta t \):

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*This research was supported by grants from the German BMBF Science Ministry, its JCHP-FFE program at COSY and the US DOE.
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Beam Dynamics and EM Fields

Dynamics 01: Beam Optics (lattices, correction, transport)
the parameter \( \tilde{\eta} \) is the limiting spin-flip efficiency.

In our spin-flipping studies, we first used Eq. (2) to determine the approximate frequency of this spin depolarizing resonance \( f_\nu = f_r(1 + G) = 917.0 \) kHz. We then experimentally determined \( f_r \) with high precision by running the rf dipole at different fixed frequencies near 917 kHz.

We next flipped the deuteron beam by linearly ramping the rf dipole’s frequency from \( f_r - 0.1 \) to \( f_r + 0.1 \) kHz, with various ramp times \( \Delta t \), and measured the deuteron polarizations after each frequency ramp as shown in Fig. 1. The curves in Fig. 1 are fits of the vector and tensor data to Eqs. (3) and (4), respectively. Note the interesting behavior of the tensor polarization, which is well-described by Eq. (4).

After optimizing \( \Delta t \) and \( \Delta f' \) for the maximum spin-flip efficiency at our maximum \( |B \cdot dl| \), we more precisely determined the spin-flip efficiencies by simultaneously measuring, after \( n \) frequency sweeps, the vector \( P_{\nu}^0 \) and tensor \( P_{\nu}^T \) polarizations. We fit these data using

\[
P_{\nu}^0/P_{\nu}^0 = (-\tilde{\eta}_V)^n, \quad P_{\nu}^T/P_{\nu}^T = \left[ \frac{3}{2}(-\tilde{\eta}_T)^2 - \frac{1}{2} \right]^n.
\]

Figure 2: The measured vector deuteron polarization ratios at 1.85 GeV/c are plotted against the number of frequency sweeps. The rf dipole’s frequency ramp time \( \Delta t \) was 60 s; its frequency half-range \( \Delta f'/2 \) was 75 Hz, and its \( |B \cdot dl| \) was 0.60 T-mm rms. The line is a fit using Eq. (5).

**Chao Matrix Formalism**

The Froissart-Stora (F-S) formula [11] has been widely used to calculate a beam’s polarization after crossing a spin resonance. However, it is valid only for a constant-rate linear crossing from far below to far above the resonance. Chao’s matrix formalism was proposed [12] to deal with conditions where the F-S formula is not valid. The Chao formalism can be used to calculate the spin dynamics anywhere inside a piecewise linear resonance crossing. It allows one to analytically solve the spin equation of motion near an isolated spin resonance, if its crossing can be expressed as a series of linear segments. Each segment must have a fixed or linearly changing distance between the spin tune \( s \) and the resonance tune \( k_r \).

After obtaining, for each segment, the time-dependent matrix describing a spinor’s evolution in the segment, one multiplies these matrices sequentially to obtain the final polarization \( P_f \).
To experimentally verify the validity of the Chao formalism, we first obtained the rf solenoid’s strength $\varepsilon$ by measuring the polarization after ramping its frequency through the resonance with various ramp times $\Delta t$ with its frequency range $\Delta f$ and voltage fixed. We then fit these data to the Froissart-Stora formula [11] Eq. (3) to obtain the measured value of $\varepsilon = (1.060 \pm 0.005) \times 10^{-5}$.

To study the Chao formalism’s predicted dependence on the beam’s momentum spread $\Delta p/p$, we varied the COSY electron cooler’s on-time at injection. It cooled the deuterons' emittances both longitudinally and transversely for 15 or 25 s. The deuterons were then accelerated to 1.85 GeV/c. The rf acceleration cavity was off and shorted during COSY’s flat top; thus, there were no synchrotron oscillations.

We tested the Chao formalism by ramping the rf solenoid’s frequency over a range $f$, which started at a frequency $f_{\text{start}}$ (well outside the rf spin resonance centered at $f_r$) and ended at a frequency $f_{\text{end}}$ near or inside the resonance, as illustrated in Fig. 3. Both $\Delta f$ and the ramp time $\Delta t$ were held fixed at 400 Hz and 100 ms, respectively, while $f_{\text{end}}$ was set to different values. After $f_{\text{rf}}$ reached $f_{\text{end}}$, the rf solenoid was turned off abruptly (in a few $\mu$s) to preserve the vertical polarization at that instant. We then measured the deuterons’ vector polarization in all polarization states. The resulting final vector polarization $P_v$ for each nonzero spin state is plotted vs $f_{\text{end}}$ in Figs. 4 and 5 for electron cooling times of 15 and 25 s, respectively.

We first calculated [8-10,12] the Chao formalism’s prediction for $\Delta p/p$ using $\Delta f$ of 400 Hz, $\Delta t$ of 100 ms, and our measured $\varepsilon$ of $1.06 \times 10^{-5}$. To take into account the beam’s momentum spread $\Delta p/p$, we next folded this result together with Gaussians representing different values of the beam’s $f_r$ spread $\delta f_{\text{SP}}$ due to $\Delta p/p$. We then fit the data in Figs. 4 and 5 with $f_r$ and $\delta f_{\text{SP}}$ as the two free parameters. The Chao formalism fits are shown as solid lines for each nonzero spin state in each figure.

We calculated $\chi^2/(N - 2)$ for each fit to compare its agreement with the data for each of the four nonzero spin states. Each $\chi^2$ analysis included only the data’s statistical errors and ignored systematic errors; nevertheless, all $\chi^2/(N - 2)$ were near 1 despite the curves’ complex shapes. The oscillations’ positions and magnitudes are very sensitive to the values of $f_r$ and $\delta f_{\text{SP}}$, respectively. As predicted, the oscillation amplitude increased as $\delta f_{\text{SP}}$ decreased. An excellent agreement of the data with the calculations in both Figs. 4 and 5 confirms the validity the Chao formalism [8-10,12].
Kondratenko Crossing

Kondratenko [13] proposed a technique for overcoming medium-strength depolarizing resonances, as an alternative to simple Fast Crossing (FC). This method, Kondratenko Crossing (KC), uses a more complicated crossing pattern, illustrated in Fig. 6, in which depolarizing phases before the resonance point are canceled by phases after the resonance. For a given maximum crossing rate, this pattern should result in less depolarization than FC. Our tests of this method used an rf solenoid to produce the resonance; thus, Fig. 6 is presented as frequency vs time; in other uses of KC the vertical axis might be the betatron tune or spin tune.

We experimentally tested KC using a 1.85 GeV/c stored polarized deuteron beam at COSY by ramping an rf solenoid’s frequency with the patterns shown in Fig. 6. We used Kondratenko’s optimization procedure [14] along with the measured resonance strength ε of (1.067±0.003)×10⁻⁵ and the previously-measured [9] resonance frequency spread δf of 23±1 Hz to calculate the optimal values of the KC pattern’s parameters defined in Fig. 6. The parameters chosen were ∆f'ₙₙ = 185 Hz, ∆f'ₙₙ = 12 ms, ∆f'ₙₙ = 400 Hz, and ∆f'ₙₙ = 160 ms. We used Chao’s matrix formalism [8-10,12] with these parameters to predict the polarization’s behavior for unbunched beam. The rf solenoid’s frequency was then programmed to form the KC pattern. To test the predicted behavior experimentally, we varied each parameter around its predicted optimal value.

We first checked that the KC pattern’s central frequency f_KC was centered on the resonance frequency f_r by varying f_KC. The resulting data are plotted in Fig. 7 for both bunched and unbunched beam and for both KC and Fast Crossing (FC). For KC, bunching shifted the peak’s central frequency by 5 Hz relative to unbunched KC; moreover, the bunched KC data had a broader flat-top. We fit the KC unbunched data to obtain f_r of 916 999.1±0.1 Hz and δf of 24.4±0.2 Hz. These values were used to predict the unbunched behavior as the parameters ∆f'ₙₙ, ∆f'ₙₙ, ∆f'ₙₙ, and ∆f'ₙₙ were individually varied; the resulting predictions and data are shown in Figs. 8-10.
Figure 10: (a) Measured 1.85 GeV/c deuteron $P_V/P_V^0$ plotted vs slow frequency ramp time $\Delta t_{slow}$. (b) Measured 1.85 GeV/c deuteron $P_V/P_V^0$ plotted vs slow frequency ramp range $\Delta f_{slow}$.

![Graph](image.png)

Figure 11: Summary of depolarization at each KC peak in Figs. 7-10 for both KC and FC, with both bunched and unbunched beam.

The depolarization values $(1 - P_V/P_V^0)$ at the KC peaks in Figs. 7-10 are summarized in Fig. 11. With the optimized KC parameters, the average measured depolarizations were $3.3 \pm 0.3\%$ and $0.8 \pm 0.3\%$ for unbunched and bunched beams, respectively; the average measured FC depolarizations were $15.6 \pm 0.2\%$ and $15.0 \pm 0.3\%$, respectively. Thus, KC reduced the depolarization far more than FC: by factors of $4.7 \pm 0.3$ and $19 \pm 1.2$ for unbunched and bunched beams, respectively.

While the Chao formalism cannot yet calculate the KC behavior for bunched beams, the measured 19-fold reduction in depolarization for KC over FC at the same crossing rate shows that Kondratenko Crossing may be quite valuable for the bunched beams used in accelerators.

Kondratenko later proposed [15] an improved version of Kondratenko Crossing to avoid most polarization loss when crossing a spin resonance. In comparison with fast crossing at the same crossing rate, KC reduced the depolarization by factors of $4.7 \pm 0.3$ and $19 \pm 1.2$ for unbunched and bunched beams, respectively.

ACKNOWLEDGMENTS


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