Transverse Schottky noise with space charge

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Schottky noise measurements
Low vs. high intensity beams

Longitudinal Schottky band at \( f_n = nf_0 \) (51 MHz)

Sum signal \( U_{\Sigma} \): beam current fluctuations \( \Delta I \)

Difference signal \( U_{\Delta} \): beam ‘offset’ fluctuations \( \Delta x \)

Spectrum analyzer

Effect of space charge on transverse Schottky bands?

set-up in the SIS-18 heavy ion synchrotron at GSI

CERN/SPS measurement (Linnecar, PAC1981)
Transverse Schottky spectrum with space charge
theory/simulation for **coasting beams**

**Space charge tune shift:**

\[ Q = Q_0 - \Delta Q_{sc} \text{ with } \Delta Q_{sc} \propto \frac{q^2 NR}{m_0 \beta_0^2 \gamma_0^3 \varepsilon} \]

Transverse equation of motion (‘rigid beam’):

\[ \frac{d^2 x}{dt^2} + Q_0^2 \omega_0^2 x - 2\omega_0^2 Q_0 \Delta Q_{sc} (x - \bar{x}) = 0 \]

**Gaussian momentum distribution:**

\[ f(\delta) = \frac{1}{\sqrt{2\pi \delta_{rms}}} \exp \left( -\frac{\delta^2}{2\delta_{rms}^2} \right) \]

**Chromatic tune spread:** \( \delta Q_{\xi} = S \delta_{\text{RMS}} \)

Dispersion function for dipole oscillations:

\[ D(u) = U_{sc} \int_{-\infty}^{\infty} \frac{f(\delta) d\delta}{u - \delta} = i \sqrt{\frac{\pi}{2}} U_{sc} w(u / \sqrt{2}) \]

with the normalized tune

\[ u = \frac{Q_f - Q_0 + \Delta Q_{sc}}{\delta Q_{\xi}} \]

**Space charge parameter:**

\[ U_{sc} = \frac{\Delta Q_{sc}}{\delta Q_{\xi}} \]

**Modified transverse Schottky band:**

\[ S(Q_f) = \frac{d(\bar{x}I)^2}{dQ_f} = \frac{S_0(u)}{|1 - D(u)|^2} \]


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Fit of the measured Schottky band to:

\[ S(f) = \frac{A \exp\left(-u^2\right)}{\left|1 - BU_{sc} w(u)\right|^2} \]

Measured transverse (lower) Schottky side-band for different numbers of ions in the ring N.

Beam parameter: \( \text{Ar}^{18+}, 11.4 \text{ MeV/u}, f_0=215 \text{ kHz} \)

Result of the fit:

<table>
<thead>
<tr>
<th>(N/10^9)</th>
<th>(\delta_{\text{rms}} \times 10^{-4})</th>
<th>(\delta Q)</th>
<th>(\Delta Q_{sc})</th>
<th>(U_{sc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>2.8</td>
<td>0.013</td>
<td>0.0</td>
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<td>3.9</td>
<td>6.7</td>
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<td>11.0</td>
<td>7.8</td>
<td>0.031</td>
<td>0.06</td>
<td>1.84</td>
</tr>
</tbody>
</table>

The space charge tune shift \(\Delta Q_{sc}\) and the space charge parameter \(U_{sc}\) can be obtained from the measured Schottky spectrum using a fit to the analytic expression \(S(f)\).
Transverse Schottky noise from \textbf{bunched beams} 
\textit{low intensity}

Transverse Schottky band at $Q_n = (n \pm Q_0)$

CERN/SPS measurement (Linnecar, PAC 1981)

Synchrotron satellites (synchrotron tune $Q_s$):

$Q_{n,k} = (n \pm Q_0) + kQ_s$

Tune spread in a rf bucket (bunch half-length $\phi_m$):

$\delta Q \approx Q_s \frac{\phi_m^2}{16} \ll S\delta_{\text{RMS}}$

Simulation time: $T = 10 \frac{Q_s}{f_0}$ (10 synchrotron periods)

Transverse ‘offset’ noise from the particle tracking code PATRIC. Bunching factor $B_i=0.35$. 

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Head-tail modes with space charge
Barrier airbag distribution

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Longitudinal bunch distribution (velocity $v_m$):
$$ f(v, \phi) = A\left[ \delta(v_m - v) + \delta(v_m + v) \right] $$

Synchrotron tune:
$$ Q_s = \frac{1}{\int_0^{2l} v_m} \text{ (bunch length $l$)} $$

head-tail tune shifts:
$$ \Delta Q_k = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc}}{2}\right)^2 + (kQ_s)^2} $$

Strong space charge $\Delta Q_{sc} >> Q_s$

positive $k$: $\Delta Q_k = \frac{(kQ_s)^2}{\Delta Q_{sc}} \rightarrow 0$
dipole moment dominates
$$ x_k = \frac{x_{+k} + x_{-k}}{2} = \cos(k \pi \phi / \phi_m) $$

negative $k$: $\Delta Q_k = -\Delta Q_{sc}$
quadrupole moment dominates
$$ x_k = (x_{+k} - x_{-k}) = (i\Delta Q_k / kQ_s) \sin(k \pi \phi / \phi_m) $$
'Offset ' noise spectrum from the particle tracking code PATRIC with a 2.5D self-consistent space charge solver.

- Good agreement with the analytic $\Delta Q_s$ for moderate space charge (code validation).
- Satellites with **negative** $k$ disappear for strong space charge.
Transverse simulation noise spectrum with space charge

Elliptic bunch distribution

\[ \Delta Q_{sc}(\phi) = \Delta Q_{sc}^{\max} \left(1 - \frac{\phi^2}{\phi_m^2}\right) \]

\[ \phi \]

Head-tail modes from the airbag model (analytic) vs. parabolic bunch (simulation noise satellites)

‘Offset ’ noise spectrum from the particle tracking code PATRIC with a 2.5D self-consistent space charge solver

\[ \frac{\Delta Q_{sc}}{Q_s} = 1.5 \]

Simulation studies for moderately strong space charge

Positive k: good agreement with airbag model

Negative k: satellites are strongly damped with space charge.

Measure the space charge tune shift in bunches

\[ \Delta Q_{sc} = \left(\frac{kQ_s}{\Delta Q_k}\right)^2 \]
Conclusions

Transverse Schottky noise spectrum from coasting beams:

- For moderately strong space charge we find a very good agreement between measured transverse Schottky bands and the analytic expression for the fluctuation spectrum of ‘rigid’ dipole oscillations.
- From a fit to the expression one can retrieve the space charge tune shift and the Landau damping rate.

Transverse Schottky noise spectrum from bunched beams:

- We obtain good agreement between the positive k synchrotron satellites from the simulation noise spectrum and the analytic head-tail tune shifts from the barrier airbag model.
- Negative k satellites are strongly suppressed due to the effect of space charge.

Possible application: Measure space charge tune shift $\Delta Q_{sc}$ from the positive k Schottky noise satellites $\Delta Q_k$. 