Purpose

• To measure emittances and Twiss parameters easily like as daily (routine) operation.
• Using profile monitors is difficult to gather all signal elements for calculating r.m.s. beam size, because of signal saturation or reduction to noise level.

=> Use six-electrode BPMs and measure second-order relative moment.

Note:
Second-order relative moment $P_{g2}$ is difference of square of r.m.s. beam size, $P_{g2} = \sigma_H^2 - \sigma_V^2$, and easily derived from signal difference.
Contents of this talk

- Higher-order moment correction for signal difference.
- Effective aperture radius.
- Entire Calibration.
- Emittance measurement by Q-scan method.
Higer-order moment correction for signal difference

- Correction: To minimize moment calculation error. Roughly \(-1 < \Delta P_{g2} < 1\) [mm²] \((-X < P_1, Q_1 < X\) [mm])
- Definition of mth-order signal difference: \(C_m, S_m\).

\(V_d\): Voltage from dth-electrode

\[
C_1 = \frac{V_1 - V_3 - V_4 + V_6}{V_1 + V_3 + V_4 + V_6}
\]
For horizontal position \(P_1\)

\[
S_1 = \frac{V_1 + V_3 - V_4 - V_6}{V_1 + V_3 + V_4 + V_6}
\]
For vertical position \(Q_1\)
Higer-order moment correction for signal difference

- Without correction (2012)

\[ C_1 \approx \frac{2}{R_{C1P1}} P_1 \]
\[ S_1 \approx \frac{2}{R_{S1Q1}} Q_1 \]

Effective aperture radius
\[ R_{C1P1} : 18.688 \text{ [mm]} \]
\[ R_{S1Q1} : 32.368 \text{ [mm]} \]

Roughly \(-1 < \Delta P_{g2} < 1 \text{ [mm}^2]\) \((-1 < P_1, Q_1 < 1 \text{ [mm]})\)
Higer-order moment correction for signal difference

- With up to third-order moments (2013)

nth-order absolute moments : $P_n, Q_n$.

\[
C_1 \approx \frac{2P_1}{R_{C1P1}} \left(1 - \frac{2P_2}{R_{C1P2}^2}\right) + \frac{2P_3}{R_{C1P3}^3}
\]

\[
S_1 \approx \frac{2Q_1}{R_{S1Q1}} \left(1 - \frac{2P_2}{R_{S1P2}^2}\right) + \frac{2Q_3}{R_{S1Q3}^3}
\]

Effective aperture radius
- $R_{C1P2} : 23.155$ [mm]
- $R_{C1P3} : \text{infinity}$ [mm]
- $R_{S1P2} : 23.155$ [mm]
- $R_{S1Q3} : 16.570$ [mm]

Roughly $-1 < \Delta P_{g2} < 1$ [mm$^2$] \((-3 < P_1, Q_1 < 3$ [mm])
Higher-order moment correction for signal difference

- With up to fifth-order moments (2014)

\[
C_1 \approx \frac{2P_1}{R_{C1P1}} \left( 1 - \frac{2P_2}{R^2_{C1P2}} + \frac{4P^2_2}{R^4_{C1P2}} + \frac{2P_4}{R^4_{C1P4}} \right) + \frac{2P_3}{R^3_{C1P3}} - \frac{2P_5}{R^5_{C1P5}}
\]

\[
S_1 \approx \frac{2Q_1}{R_{S1Q1}} \left( 1 - \frac{2P_2}{R^2_{S1P2}} + \frac{4P^2_2}{R^4_{S1P2}} + \frac{2P_4}{R^4_{S1P4}} \right) + \frac{2Q_3}{R^3_{S1Q3}} + \frac{2Q_5}{R^5_{S1Q5}}
\]

Effective aperture radius

\[
R_{C1P4} : 19.953 \text{ [mm]}
\]

\[
R_{C1P5} : 17.499 \text{ [mm]}
\]

\[
R_{S1P4} : 19.953 \text{ [mm]}
\]

\[
R_{S1Q5} : 19.531 \text{ [mm]}
\]

Roughly \(-1 < \Delta P_{g2} < 1 \text{ [mm}^2\text{]} \) \((-5 < P_1, Q_1 < 5 \text{ [mm]})\)
Entire Calibration

• The concept was presented at LINAC2012.

• **Absolute moment** was **changed** by steering magnet.

• **Relative moment** was **not changed** by steering magnet.

• We deduce the **relative attenuation factors** between electrode channels.

Red circles: Scanned beam positions $P_1$, $Q_1$
• Typical measured moments in entire calibration.
Entire Calibration
• Typical measured moments in entire calibration.
Emittance Measurement by Q-scan

Six-Electrode Beam Position Monitor (6EBPM) Quadrupole Magnet (QM)

Steering Magnet (ST)

Twiss Parameter Reference Point

Equipment Layout

Variation of Q-mag Currents

Measured Second-Order Relative Moments P_{g2,M}
Deduced Emittances and Twiss Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ $[\pi \text{mm} \cdot \text{mrad}]$</td>
<td>$0.168 \pm 0.002$</td>
<td>$0.299 \pm 0.001$</td>
</tr>
<tr>
<td>$\beta$ $[\text{m}]$</td>
<td>$14.7 \pm 0.1$</td>
<td>$5.7 \pm 0.2$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2.25 \pm 0.04$</td>
<td>$0.50 \pm 0.03$</td>
</tr>
</tbody>
</table>

All $P_{g2,C}$ and $P_{g2,M}$ are agree within error bars.

Twiss Parameter Reference Point

$P_{g2,C} = \sigma_{H,C}^2 - \sigma_{V,C}^2$
Emittance Measurement by Q-scan

**Graph 4:**
- $I_{SET} = (-34, 19.5, -3, 3) \; [A]
- Beam Size $\sigma$ vs Distance from PM_LS_1 [m]
- 2nd-Order Relative Moment $P_{g2}$ vs Distance from PM_LS_1 [m]

**Graph 5:**
- $I_{SET} = (-42, 26, -4, 4) \; [A]
- Beam Size $\sigma$ vs Distance from PM_LS_1 [m]
- 2nd-Order Relative Moment $P_{g2}$ vs Distance from PM_LS_1 [m]

**Graph 6:**
- $I_{SET} = (-50, 32.5, -5, 5) \; [A]
- Beam Size $\sigma$ vs Distance from PM_LS_1 [m]
- 2nd-Order Relative Moment $P_{g2}$ vs Distance from PM_LS_1 [m]

**Graph 7:**
- $I_{SET} = (-58, 39, -6, 6) \; [A]
- Beam Size $\sigma$ vs Distance from PM_LS_1 [m]
- 2nd-Order Relative Moment $P_{g2}$ vs Distance from PM_LS_1 [m]
**Emittance Measurement by Q-scan**

- **First Panel:**
  - Beam size $I_{SET} = (-66, 45.5, -7.7) [A]$
  - Properties at $H_C$ and $V_C$
  - nd-Order Relative Moment $P_{g2}$

- **Second Panel:**
  - Beam size $I_{SET} = (-74, 52, -8.8) [A]$
  - Properties at $H_C$ and $V_C$
  - nd-Order Relative Moment $P_{g2}$

- **Third Panel:**
  - Beam size $I_{SET} = (-82, 58.5, -9.9) [A]$
  - Properties at $H_C$ and $V_C$
  - nd-Order Relative Moment $P_{g2}$

- **Fourth Panel:**
  - Beam size $I_{SET} = (-90, 65, -10.10) [A]$
  - Properties at $H_C$ and $V_C$
  - nd-Order Relative Moment $P_{g2}$