Fast development of the technique of photo-cathode electron photoinjectors has resulted in creation of compact and accessible sources of relativistic (several TeV) dense (“pinch”) electron bunches. Methods for decrease of the energy spread (cooling) are actual from the point of view of various applications of such beams, including free-electron lasers (FEL). However, cooling methods are developed now basically for electron beams of significantly higher energies (petawatt) in high-energy density (high-energy density) short electron bunches, the strong Coulomb interaction of the particles results in a significant spread of the accelerated flow of cooling.

In this letter, we propose to provide cooling by the use of electron cooling in a circular polarized “cooling” undulator at the presence of an axial magnetic field, (Fig. 1). If the beam energy is enough to provide the resonant cooling in the undulator, \( \gamma = \Theta_{z} / \Delta \gamma \), where \( \gamma \) is the electron axial velocity, \( \Theta_{z} \) is a wavelength, and \( \Delta \gamma \) is the relativistic electron mass factor. In this situation, the velocity of undulator oscillations, \( \Theta_{z} \), depends strongly on the initial axial velocity.

Non-radiative “axial” cooling is based on the fact that the axial velocity spread is the only factor important for the FEL operation. This spread can be decreased due to its “transformation” into the spread in the velocity of electron rotation in the cooling system. Electrons move along axial magnetic field and enter the cooling undulator with the adiabatically growing field in the input section, where each electron gets its own rotatory velocity (Fig. 1a). If at the input of the system every particle possesses only the axial velocity \( \gamma = \Theta_{z} / \Delta \gamma \), then the axial velocity in the regular region of the undulator is determined by the energy conservation law: \( \gamma = \Theta_{z} / \Delta \gamma \). Thus, the initial axial velocity excess, \( \Delta \gamma \), is compensated by the greater rotatory velocity, \( V_{\perp} \).

If such a cooling system is used in a FEL, then the overall energy of the electron is determined by the formula of the resonance situation, when it is difficult to provide the adiabatically smooth entrance into the undulator. Instead of \( \gamma = \Theta_{z} / \Delta \gamma \), the spread in \( \gamma \) disappears. This condition is independent of the initial spread, \( \gamma_{0} \). Evidently, we should use the range of parameters, where \( \gamma_{\perp} / \gamma_{0} > \gamma_{0} \), so that the initial axial velocity excess, \( \Delta \gamma \), is compensated by the greater rotatory velocity, \( V_{\perp} \).

Figure 2. Output axial spread versus the initial spread at various initial speeds in the rotary velocity \( V_{\perp}. \)

Let us consider the condition for the transverse momentum of a particle moving in the axial uniform magnetic field, \( B_{0} \), and in the quasi-periodical field of the undulator, \( \beta_{0} \), \( \gamma_{0} \), \( \Theta_{z} \), \( \Delta \gamma \), and the electron velocity in the initial section of the undulator.

\[
\beta_{0} \gamma_{0} = \gamma_{0} = \Theta_{z} / \Delta \gamma,
\]

where the velocity of electron rotation in the undulator field, \( \beta_{0} = \Theta_{z} / \Delta \gamma \), depends on the mismatch between the electron cyclotron frequency and the bounce-frequency of electron oscillations in the undulator, \( \Delta \gamma = \Theta_{z} / \Delta \gamma \). Thus, the initial axial velocity is related to the axial velocity in the regular region of the undulator as follows:

\[
\beta_{0} \gamma_{0} = \gamma_{0} = \Theta_{z} / \Delta \gamma.
\]

Thus, the initial spread in \( \gamma \) is decreased due to the increase in the transverse electron velocities. However, \( \beta_{0} \) depends on \( \gamma_{0} \) weakly in the optimal cooling regime, \( \beta_{0} \sim \Delta \gamma / \gamma_{0} \). At the same time, the undulator factor is related by Eq. (3) and (4). Evidently, we should choose the electron transverse momentum, \( \gamma_{0} = \Theta_{z} / \Delta \gamma \), and the spread in the transverse velocity (4).

\[
\Delta \gamma = \Theta_{z} / \Delta \gamma
\]

Second, there is the spread in the transverse electron position. As the undulator field is not uniform, this spread induces the corresponding spread in the undulator velocity so that:

\[
\Delta \gamma = \Theta_{z} / \Delta \gamma
\]

The third source of the uncompensated spread is the non-ideal transformation of the axial spread described by the non-zero \( \gamma_{0} \) in Eq. (5):

\[
\Delta \gamma = \Theta_{z} / \Delta \gamma
\]

Let us notice that an increase in the undulator parameter leads to the reduction in the Doppler frequency up-conversion factor due to the increase in the transverse electron velocity. However, \( \beta_{0} \) depends on \( \gamma_{0} \) weakly in the optimal cooling regime, \( \beta_{0} \sim \gamma_{0} / \gamma_{0} \). At the same time, the undulator factor is related by Eq. (6) to the mismatch between the cyclotron undulator frequencies, \( \gamma_{0} = \Theta_{z} / \Delta \gamma \). Evidently, it should be great enough to avoid the close-to-resonance situation, when it is difficult to provide the adiabatically smooth entrance into the undulator. In addition, in the case of a strong system, the system is very critical to the initial spread.

Let us consider a 5 MeV electron bunch with the parameters typical for modern photo-injectors: energy spread \( \Delta \gamma / \gamma_{0} \), normalized emittance \( \epsilon_{0} \), normalized brightness \( B_{0} \), and the bending radius in the case of a cooling undulator with \( \gamma_{0} / \Delta \gamma = \gamma_{0} \). (100 mrad mm m 120 u ~ K 0c c / \lambda L 20)cm~ (100 MeV ~ \lambda L 20 cm~ 120 u ~ K 0c c / \lambda L 20)cm~

Figure 3 illustrates the electron motion in the non-optimal region of the cooling system. The increase in \( \beta_{c} \) leads to a decrease in the averaged axial velocity, so that \( \beta_{c} / \beta_{0} \). This is accompanied with the decrease in the axial velocity spread down to \( \Delta \gamma / \gamma_{0} \), and with the decrease in the spread in the transverse velocity. In the ideal case, when \( \gamma_{\perp} / \gamma_{0} = \Delta \gamma / \gamma_{0} \), decreases with the decrease in \( \gamma_{0} \). (least curve in Fig. 2).

Figure 3. a) Averaged axial gamma-factor and spread in the axial gamma-factors versus the axial coordinate in the non-optimal region of the undulator. b) Evolution of the averaged transverse velocity and axial velocities of different electrons.

In the suggested non-radiative cooling scheme of ~5% spread velocity of 5 MeV electron beam (typical for photo-injectors) can be reduced to ~0.05% at distances as long as 20 undulator periods. The described principle works also for high energy beams (100 MeV and more), where RF undulator instead of DC-magnet undulator is necessary.