A REVIEW OF EMITTANCE EXCHANGER BEAMLINES: PAST EXPERIMENTS AND FUTURE PROPOSALS

J. Thangaraj†, Fermi National Accelerator Laboratory, Batavia, IL, USA

Abstract
Emittance exchangers (EEX) are advanced phase space manipulation schemes where the transverse phase space of the electron beam is exchanged with the longitudinal phase space. The first experimentally demonstrated concept of the emittance exchange at the A0 photoinjector at Fermilab used a transverse deflecting cavity (TDC) sandwiched between two doglegs. This paper briefly reviews the history of the emittance exchange beamline experiments from a low charge beam without RF chirp to a high charge beam with RF chirp including collective effects such as coherent synchrotron radiation. The paper concludes by discussing future emittance-exchange schemes that have been proposed and proposes two additional schemes that can be implemented in existing modern linacs. As an example, we present an improved emittance exchanger scheme that uses a TDC sandwiched between two chicanes. The significant advantage of this scheme is that it allows the use of the expensive transverse deflecting cavity for diagnostics and still allows the flexibility to use the existing beamline either as a bunch compressor or an emittance exchanger.

INTRODUCTION
More than a decade ago, a scheme for exchanging the transverse phase space of the beam with the longitudinal phase space of the beam in a linac was proposed [1]. The original scheme proposed using a transverse deflecting mode cavity placed at the center of a four-dipole chicane. In this scheme, the exchange was not exact: some residual coupling existed even for an ideal zero-length cavity. Another configuration that did an exact exchange of the emittance was then suggested in [2]. This is shown in Fig. 1. In this scheme, the deflecting mode cavity was sandwiched between two doglegs assuming an ideal deflecting mode cavity i.e. the thick-lens effect was not taken into account. The final rms emittances after the emittance exchange beam line can be written as follows:

\[ e_{x,\text{out}}^2 = e_{z}^2 + \left( \frac{\lambda x}{4D} \right)^2 (\lambda x)^2 + \alpha^2 D^2 (\delta z)^2 + 2\alpha D (\delta z) \]

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where \( \alpha D \) is the longitudinal dispersion, \( x', z, \) and \( \delta \) are the incoming x-angle, longitudinal position, and energy spread of the beam, respectively, \( \lambda \) is the wavelength of the cavity and \( D \) is the dispersion of a single dogleg. In a practical emittance exchange beamline, the thick lens effect of the cavity is always present and so the outgoing emittance is coupled with the incoming emittance to some degree. An experimental implementation of such a scheme was done at the A0 photoinjector facility at 15 MeV [3]. The experiment showed near-ideal exchange at a lower charge. At higher charges, other effects such as the coherent synchrotron radiation (CSR) effects, and space-charge effects start becoming important. CSR studies of the emittance-exchange beamline were reported in [4]. The study pointed out that the bunch length of the beam after the emittance exchange decreased when operated with an RF chirp. Note, this is not only due to the matching of the longitudinal dispersion but also due to the compensation of the thick-lens effect of the deflecting mode cavity. We briefly mention the R-matrix formalism to understand the advantage of the chirped-beam.

Chirping to Reduce the Thick-Lens Effect on the Cavity

The complete R-matrix \((x, x', z, \delta)\) of the emittance exchanger, including the finite length of the cavity, can be written as follows:

\[
\begin{pmatrix}
0 & \frac{Lc}{4} & -\frac{(4L+Lc)}{4\eta} & \eta - \frac{4L+Lc}{4}
0 & 0 & -\frac{1}{\eta} & -\alpha
-\alpha & \eta - \frac{4L+Lc}{4} & \frac{\alpha Lc}{4\eta} & \frac{a^2 Lc}{4}\n-\frac{1}{\eta} & \frac{(4L+Lc)}{4\eta} & \frac{\alpha Lc}{4\eta} & \frac{a^2 Lc}{4}\end{pmatrix}
\]

where \( \alpha \) is the bending angle of the dipoles, \( Lc \) is the length of the deflecting cavity, \( \eta \) is the dispersion of a dogleg, and \( L \) is the length of the dogleg. When the RF-chirp is set to \( \frac{1}{\eta} \), the R-matrix is instead written as:

\[
\begin{pmatrix}
0 & \frac{Lc}{4} & -\frac{1}{\alpha} & \eta - \frac{4L+Lc}{4}
0 & 0 & 0 & -\alpha
-\alpha & \eta - \frac{4L+Lc}{4} & 0 & \frac{a^2 Lc}{4}\n-\frac{1}{\eta} & \frac{(4L+Lc)}{4\eta} & \frac{\alpha Lc}{4\eta} & \frac{a^2 Lc}{4}\end{pmatrix}
\]

As can be seen in the above matrix, both the \( R_{33} \) and \( R_{43} \) elements are reduced to zero from the original matrix due to the chirp. To first order, this leads to a reduction in the bunch length and the energy spread after the emittance exchanger. In the transverse plane both the \( R_{13} \) and \( R_{23} \) terms are also reduced due to the chirp indicating possible reduction in beam size and beam divergence as well. A positive side-effect of using a chirped beam is the decrease in the \( x' \) after the cavity. Recall that after the cavity, \( \Delta x' = \kappa \Delta z \); where \( \kappa \) is the strength of the deflecting cavity set to \( \frac{1}{\eta} \) for EEX. So if we reduce the bunch length at the cavity by adding energy chirp, the beam divergence after the cavity is also reduced.
thus reducing the emittance growth from second order dispersive aberration. A negative side effect of using a chirped beam is the increase in coherent radiation effects that can spoil the transverse emittance of the beam in the bend plane. An experimental study of running the emittance exchange experiment at higher charge with an energy-chirped beam showed good agreement with simulation. The configuration of such a beamline is shown in Fig. 1 and the experimental results are shown in Fig. 2. In other words, chirping the beam compensates the thick-lens effect of the cavity [5] and improves the exchange ratio. Emittance dilution still exists possibly due to space-charge and second order effects.

Figure 1: (a) Schematic of the classic double-dogleg emittance exchanger (b) Booster cavity before EEX to chirp and compensate thick-lens effect (c) Accelerating cavity inside the EEX to overcome thick-lens effect (d) Twin deflecting cavities to compensate thick-lens effect

Figure 2: Measured value of the emittance exchange ratio and their products. As the RF-chirp increases, the ratio tends towards one. There is still some emittance dilution possibly due to second order effects and space charge

Since thick-lens effect due to the transverse deflecting mode cavity caused a coupling in the exchange, initial efforts were concentrated on overcoming this effect. One way to overcome the thick-lens effect of the TM110 cavity is to use an accelerating-mode cavity before or after the deflecting cavity as shown in [6]. By appropriately tuning the chirp of the accelerating mode cavity, the thick-lens effect can be made zero. Alternatively, we can use another TM110 cavity to do the same. In this case, the strength of the deflecting mode cavity must be changed a bit but the thick-lens effect can be made zero. In most of the works listed above, EEX has a baseline design that uses dogleg geometry with no ‘embedded’ quads inside the dogleg. A negative-drift based EEX was proposed that could be used as a bunch compressor that has significant benefit of compressing without a RF-chirp (energy-phase) correlation, which saves energy by operating on-crest and is less vulnerable to CSR-effects at high beam energy [7]. A laser-assisted scheme wherein instead of the deflecting mode cavity, a TEM10 laser and an undulator is used to compensate the dispersion for some particles [8]. A generalized solution for an EEX based on a dogleg geometry has been done in [9]. It is worth pointing out that the idea of an emittance exchanger can be used in an ERL [10] for handling large fractional energy spread beam after the FEL process, for HGHG [11], THz generation [12], and longitudinal shaping for advanced accelerator applications [13, 14].

Recently, a chicane-type EEX which has the benefit of operating both as a chicane and as an EEX was proposed by introducing a negative unity transfer matrix using a quadrupole doublet before the TM110 cavity [15]. The chicane being widely available in many linear accelerators can take advantage of this scheme. One of the limitations of the EEX scheme is that the strength required by the deflecting mode cavity can be large at high energy. In order to reduce the required power on the cavity and thus reduce the cooling cost, a dispersion-boosted, in-line, chicane emittance exchanger based on dispersion boosting with an emphasis towards shaping application was done in [16]. Sometimes an emittance-exchanger which shifts the jitter from time to transverse position is not favorable and one way to overcome this is by using a double emittance exchanger configuration by ganging one EEX followed by another EEX. This way the phase-space manipulation is done between the transfer from the transverse to longitudinal and back to transverse phase space allowing for bunch compression, bunch train production, shaping etc. In the context of microbunching instability, adding a few micron thick beryllium foil after the first EEX section increases the energy spread at the end of the double emittance exchanger beamline (due to multiple scattering) [6]. This could be an alternative to the laser heater used in modern FELs.

Chicane based EEX design, while an improvement, still has a major limitation that constrains the deflecting mode cavity to be at the center of the chicane. Sometimes, this could limit the use of the transverse deflecting mode cavity for beamline diagnostics and operations. Therefore, an EEX design that has the deflecting mode cavity in line with the linac would be helpful. Ideally this will allow the beamline to operate as a chicane followed by a deflecting cavity, or being used as an EEX. This can be done by the configuration shown in Fig. 3 using a flipper EEX based on a double chicane configuration. The flipper EEX converts the chicane into a dogleg-like lattice with twice the dispersion thereby the strength needed for the cavity drops by a factor of two. By adding a magnification/demagnification lattice, more reduction is possible up to a factor of 3 or more, but the non-bending y-plane needs to be carefully managed. A
triplet to do the negative unity matrix is beneficial but space constraints might allow only a doublet. The advantage of this scheme is that existing beamlines that have two chicanes with transverse deflecting mode cavity between them can be readily converted into an EEX beamline by adding quadrupoles. There are some practical limitations to this scheme. The extra chicanes in this scheme means extra space and all the associated costs. So if space is a limitation, a compact chicane might be helpful [17]. CSR effects will still be a problem at the last few dipoles. In a real beamline, more magnets might be necessary for appropriate transverse control before and after the EEX beamline. Since the quadrupoles are inside the dispersive section, second-order effects must be taken into account in this scheme. In summary, considering that most single-pass FELs and some linacs have at least two bunch compressors, this EEX scheme could be an option.

![Diagram of Emittance Exchange Scheme Using One Deflecting Mode Cavity and Two Chicanes](image)

Figure 3: Emittance exchange scheme using one deflecting mode cavities and two chicanes. The strength of the cavity can be reduced by at least a factor of 2.

Finally, most of the emittance exchangers discussed above use configuration that involves a dispersive section followed by a transverse deflecting mode cavity that is then followed by a dispersive section. This is a cheaper scheme as it involves only one deflecting mode cavity. But typically such a beamline is coupled both to the longitudinal and the transverse dispersion of the lattice unless uncoupled by a properly tuned quadrupole lattice before and after the beamline. Another possible scheme is to have a deflecting mode cavity followed by dispersive section which is then followed by a deflecting mode cavity as shown in Fig. 4.

![Diagram of Emittance Exchange Scheme Using Two Deflecting Mode Cavities and Two Chicanes](image)

Figure 4: Emittance exchange scheme using two deflecting mode cavities and two chicanes

While this is an expensive option, such a beamline has the feature of a $\text{Spure}^T$ exchange of one phase-space variable ($xS$) to the other ($SzS$). Also, such a beamline could also be used as a reversible laser heater by rotating the deflecting mode cavity to deflect in the $y$-plane [18]. This $\text{Spure}^T$ scheme is also made robust against the thick lens effect of the transverse deflecting mode cavity by chirping the accelerating mode cavity before and after it appropriately for compensation. The deflecting mode cavity is set as follows: $\alpha_1 = \frac{1}{2\eta}; \alpha_2 = \frac{1}{2\zeta}$. The accelerating mode cavity is set as $h = \frac{1}{2\zeta}; h_1 = \frac{1}{2\zeta}$, where $\zeta$ is the longitudinal dispersion of the chicane and $h, h_1$ is the RF-chirp of the cavities respectively. The length of the first deflecting mode cavity is twice that of the second. Setting appropriately the negative drift, the final EEX matrix for this beamline is

$$
\begin{bmatrix}
0 & 0 & 0 & 2\eta \\
0 & 0 & -\frac{1}{2\eta} & 0 \\
-2\eta & 0 & 0 \\
\frac{1}{2\eta} & 0 & 0 & 0
\end{bmatrix}
$$

Realistic implementations of future proposals have to take into account the beam energy at which this experiment will be done which sets a limit on the RF power required and the always present second-order effects in all of these schemes.

**SUMMARY**

Emittance exchangers are a sub-set of the broader area of phase-space manipulation schemes. Progress in theory, experiment and simulation have spawned various ideas on using emittance-exchangers in practical accelerators, radiation sources and future advanced accelerator designs. Efforts to make the emittance-exchange scheme compact and cheap would be a good next step.

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**REFERENCES**


