THE PHYSICS PROGRAMME OF MICE STEP IV∗
V. Blackmore, University of Oxford, UK
On behalf of the MICE Collaboration

Abstract
The international Muon Ionisation Cooling Experiment (MICE) is progressing toward a full demonstration of the feasibility of the cooling technology required for neutrino physics and muon colliders. Step IV will provide the first precise measurements of emittance and determine the influence of material properties on emittance reduction. The physics programme of the Step IV measurements is described in detail, along with a longer term view to demonstrating and studying (sustainable) ionisation cooling with re-acceleration.

INTRODUCTION
The front-end of a Neutrino Factory or Muon Collider will produce muons that occupy a large area of phase space. The phase-space area, or emittance, of these muons must be reduced (i.e. “cooled”) so as to fit within the acceptance of downstream acceleration and storage systems. The expected beam produced at the front-end has a large transverse emittance, \( \epsilon_N \approx 15–20 \pi \text{ mm.rad} \), and a large momentum spread of \( \approx 20 \text{ MeV}/c \). For a Neutrino Factory, the emittance must be reduced to 2–5 \( \pi \text{ mm.rad} \) [1], whereas more stringent requirements are placed on a Muon Collider. Conventional cooling techniques are ineffective, as the muon lifetime is short, and it is necessary to develop a new method of maximising the muon flux through an accelerator.

Ionisation cooling is the only practicable approach to reducing the emittance of a muon beam. Muons are passed through a low-\( Z \) “absorber” where they lose energy by ionising the medium. This reduces the transverse phase space area of the beam. The longitudinal momentum component must be restored in RF-cavities for sustainable ionisation cooling.

MICE STEP IV
A reduction in transverse normalised emittance is approximated by the ionisation cooling equation,

\[
\frac{d\epsilon_N}{ds} \approx \frac{\epsilon_N}{\beta^2 E_\mu} \left( \frac{dE}{ds} \right) + \frac{\beta_\perp (13.6 \text{ MeV})^2}{2 \beta^2 E_\mu m_\mu X_0},
\]

(1)

where \( \epsilon_N \) is the normalised transverse emittance, \( \beta \) the relativistic velocity, \( E_\mu \) the energy, \( \frac{dE}{ds} \) the energy lost by ionization, \( m_\mu \) the mass of the muon, \( X_0 \) the radiation length of the absorber material and \( \beta_\perp \) the transverse beta function at the absorber. The first term of this equation describes “cooling” by ionisation energy loss, and the second term describes “heating” by multiple Coulomb scattering. When these terms are equal, the equilibrium emittance of the cooling channel is,

\[
\epsilon_{eq} \approx \frac{\beta_\perp (13.6 \text{ MeV})^2}{2 \beta m_\mu X_0} \left( \frac{dE}{ds} \right)^{-1}.
\]

(2)

The smaller the equilibrium emittance, the more efficient the cooling channel. For this purpose it is desirable to minimise \( \beta_\perp \) at the absorber and maximise \( X_0 \left( \frac{dE}{ds} \right) \).

The Muon Ionisation Cooling Experiment (MICE) [2] will demonstrate the performance of one “SFOFO” [3] lattice cell of the Neutrino Factory Feasibility Study 2 design [1]. The cell has a large momentum acceptance, accepting a spread of \( \approx 20 \text{ MeV}/c \) over a 140–240 \( \text{MeV}/c \) central momentum range.

Figure 1 shows the Step IV lattice section. This consists of two 4T Spectrometer Solenoids containing scintillating fibre tracker planes, either side of an Absorber Focus Coil (AFC) that can contain either liquid hydrogen (LH2) or lithium hydride (LiH) absorbers. A variable amount of high-\( Z \) material is situated at the upstream side of the upstream Spectrometer Solenoid and provides the cooling cell with a range of input emittances. Step IV will study how material and beam properties affect emittance reduction. A further AFC module and RF system will be installed prior to MICE Step V to demonstrate sustainable ionisation cooling. Progress towards the construction of Step IV is detailed in [4].

The beam delivered by the MICE muon beam line [5] is of low intensity compared to conventional primary beams, and has been characterised during Step I [6]. MICE measures the phase space components, \((x, y, p_x, p_y)\) of individual muons as they cross the scintillating fibre tracker planes in the up- and downstream Spectrometer Solenoids. A “beam” is constructed from these individually measured muons, and a covariance matrix calculated. In this way, MICE measures a change of emittance of \( \approx 5\% \) (10\% in Step V) to a relative precision of 1\%.
STEP IV PHYSICS PROGRAM

Step IV will study the reduction in emittance as a function of the inputs to Equation 1. An example is shown in Figure 2 where a beam with an initial emittance of $\varepsilon_{\text{in}} \approx 6\pi\,\text{mm.rad}$ and momentum of $p_z = 200\,\text{MeV}/c$ is propagated through a simulation of Step IV containing 350 mm LH$_2$ (black). The scintillating fibre tracker planes reduce the emittance by a small amount, but the majority of emittance reduction occurs in the absorber (centred on $z = 0$). Also shown (blue) are the reconstructed emittances up- and downstream of the absorber as would be measured by the tracker planes.

Figure 3 shows the evolution of emittance reduction in Step IV as a function of input emittance, with all other parameters kept constant, for both LH$_2$ and LiH. At small initial emittance, the beam is heated by multiple scattering. At large initial emittance, the beam is cooled by $\approx 5\%$.

The “Physics Grid”

A systematic exploration of the properties that determine the cooling performance of the MICE lattice is embodied by a $3 \times 3 \times 5 \times 3 \times 5$, $(\varepsilon_{\text{in}}, p_z, \beta_z)$ parameter space (“physics grid”). An example is shown in Table 1, where $\varepsilon_{\text{in}}$ and $p_z$ are varied for a constant magnetic lattice configuration (and hence $\beta_z$ at the absorber).

At each point, a sample of $\geq 100'000$ muons is measured by the up- and downstream tracker planes and a covariance matrix is constructed before and after the absorber. Hence the change in emittance will be measured to high precision and the transmission of the channel can be determined by comparing the proportions of the selected beam measured before and after the absorber. For optimum performance, the selected beam must be matched into the up- and downstream Spectrometer Solenoids such that $\alpha_z = 0$, $\beta_z = 333.34$ mm at $p_z = 200\,\text{MeV}/c$. The quality of this match is also encompassed within the physics grid methodology.

Step IV will study such a physics grid for three absorber configurations: empty, LH$_2$ and LiH, in two magnetic lattice modes. In Solenoid mode, the magnetic field along the cooling channel is all of the same sign, whereas in Flip mode, the field changes sign at the centre of the absorber. Changing the sign of the field at the absorber is believed to control the buildup of canonical angular momentum [8], which can adversely affect the emittance of a beam exiting a cooling channel. Hence, it is a further quantity that Step IV will study for both absorbers.

Emittance Exchange

The primary aim of MICE is to demonstrate a sustainable reduction in the transverse, normalised emittance of a NF-like muon beam. However, both transverse and longitudinal cooling are required before a beam is suitable for a Muon Collider. The longitudinal emittance can be reduced via a technique known as “emittance exchange.” A beam is passed through a dipole, acquiring a position-energy correlation (longitudinal momentum dispersion), and then a wedge-shaped absorber (Figure 4). When crossing the wedge, the position-energy correlation is removed, and an exchange of longitudinal emittance to transverse emittance occurs. Repeatedly cooling a beam in the transverse direction, interspersed with wedge absorber sections, is one method of reducing all six phase-space dimensions. Due to the single-particle nature of the MICE muon measurements, it is possible to select a beam with large position-energy correlations upstream of a wedge-shaped absorber and measure longitudinal cooling downstream.

Multiple Scattering and Energy Straggling

Although the “physics grid” allows for the determination of the equilibrium emittance of LH$_2$ and LiH, and hence the balance of cooling by ionisation and heating by multiple scattering, MICE also measures $(x, y, z, p_x, p_y, p_z)$ for each individual muon before and after the absorber. This makes measurements of multiple scattering and energy straggling distributions in both materials possible, though non-trivial [7]. A measured multiple scattering distribution would be complementary to the MuScat experiment [10].

CONCLUSIONS

MICE Step IV will measure the influence of material and beam parameters on ionisation cooling. A change in emittance of $5\%$ ($10\%$ in Step V) will be measured to a relative precision of $1\%$, as a function of initial emittance, momentum, $\beta_z$ at the absorber and absorber material. Canonical angular momentum will be studied by changing the sign of the magnetic field at the absorber.

The physics accessible to Step IV is summarised in Table 2 and compared to Step V. Following Step IV, MICE will add a further AFC module and RF cavities to the lattice, and demonstrate sustainable ionisation cooling with...
Figure 3: Simulated evolution of emittance reduction in Step IV, as a function of $\varepsilon_{\text{in}}$, at constant $p_z$ and $\beta_\perp$ [7].

Table 2: The Physics Accessible to MICE Step IV and V

<table>
<thead>
<tr>
<th>Study of properties that determine cooling performance</th>
<th>Step IV</th>
<th>Step V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material properties of LH$_2$ and LiH</td>
<td>Yes</td>
<td>LH$_2$ or LiH</td>
</tr>
<tr>
<td>$\varepsilon_{N,\perp}$ reduction</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demonstration of sustainable ionisation cooling</th>
<th>Step IV</th>
<th>Step V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\perp}$ reduction with re-acceleration</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$\varepsilon_{\perp}$ reduction and $\varepsilon_{</td>
<td></td>
<td>}$ evolution</td>
</tr>
<tr>
<td>$\varepsilon_{\perp}$ reduction, $\varepsilon_{</td>
<td></td>
<td>}$ and $L$ evolution</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT

This research is supported by grants from the Science and Technology Facilities Council (UK). MICE is under construction at the Rutherford Appleton Laboratory, UK, and the support of the ISIS staff is gratefully acknowledged.

REFERENCES


