Abstract

Periodic sequences of independently phased accelerating cavities and focusing solenoids are used in MeV and GeV energy range linacs. The beam dynamic investigation is difficult for such superconducting linear accelerator. The matrix calculation was preferably used for primary choused of accelerating structure parameters. This method does not allows properly investigate the longitudinal motion. The smooth approximation can be used to investigate the nonlinear ion beam dynamics in such accelerating structure and to calculate longitudinal and transverse acceptances. The advantages and disadvantages of each method will describe, the results of investigation will compare. The user friendly software BEAMDULAC-SCL for ion beam dynamic analysis was created. A numerical simulation of beam dynamics in the real field are carried out for the different variants of the accelerator structure based on previously analytically obtained results.

INTRODUCTION

High-current accelerators have great perspectives for problems of thermonuclear fusion, safe nuclear reactors, transmutation of radioactive wastes and free electron lasers. A large number of low energy particle accelerators are applied in micro- and nanoelectronics, material science, including the study of new construction materials for nuclear industry, in medical physics, in particular for cancer treatment by using of the accelerators of protons and light ions, in radiation technology. It is proposed to use one universal accelerator, consisting of independently phased cavities and solenoids sequence to solve these problems.

An ion superconducting linac is usually based on the superconducting (SC) independently phased cavities. This linac consists of the niobium cavities which can provide typically 1 MV or more of accelerating potential per cavity. Such structures can be used for ion acceleration with different charge-to-mass ratio in the low energy region [1] and for proton linac in the high-energy region (SNS, JHF, ESS project). It is desirable to have a constant geometry of the accelerating cavity in order to simplify manufacturing and to decrease the linac cost. Such geometry leads to a non-synchronism but a stable longitudinal particle motion can be provided by proper RF cavities phasing. The beam can be both longitudinally stable and accelerated in the whole system by control of the accelerating structure driven phase and the distance between the cavities. In this paper two methods of the beam dynamics investigation are compared for low ion velocities and for the charge-to-mass ratio $Z/A = 1/66$.

This comparison can be demonstrated as an example of a post-accelerator of radioactive ion beams (FRIB) linac, where beam velocity increases from $\beta = 0.01$ to $\beta = 0.06$ [1].

The beam transverse focusing can be provided with the help of SC solenoid lenses, following each cavity and with the help of special RF fields. As it was shown early the beam focusing can be realized for the solenoid field near $B \approx 20$ T. The value of magnetic field $B$ can be reduced by using of addition alternating phase focusing (APF). The smooth approximation has been applied to study the APF in RIB linac. By adjusting the drive phase ($\varphi_1$ and $\varphi_2$) of the two cavities, we can achieve the acceleration and the focusing by less magnitude of the magnetic field $B$ [2]. Adding a solenoid into focusing period will also allow the separate control of the transverse and longitudinal beam dynamics. A schematic plot of one period of the accelerator structure is shown in Figure 1. The low-charge-state low velocity beams require stronger transverse focusing than one is used in existing SC ion linac. Early investigation of beam dynamics shows that with the initial normalized transverse emittance $\varepsilon_T = 0.1 \pi \cdot \text{mm} \cdot \text{mrad}$ and the longitudinal emittance $\varepsilon_L = 0.3 \pi \cdot \text{keV/}u\cdot\text{nsec}$ the connection between the longitudinal and transverse motion can be neglected if maximum beam envelop $X_m < 3-4 \text{ mm}$ and inner radius of drift tubes $a = 15 \text{ mm}$.

Beam dynamics in such systems cannot be studied by means of analytical methods only. The initial setup of the system consisting of different types superconducting resonators and focusing solenoids or quadrupoles, can be performed using the transfer matrix calculation and the method of smooth approximation, and then refined the beam dynamics simulation in polyharmonic field.

Figure 1: Layout of structure period.

The BEAMDULAC-SCL code is developed in the laboratory DINUS and allows to do the comprehensive research of ion beam dynamics in a different structures that satisfy the acceleration of many methods. To evaluate the accelerator parameters implemented transfer matrix calculation method. Using a smooth averaging technique we can to determine the stability region and to calculate...
the dynamics of a single particle and beam. For completion, to verify selections, we perform the calculation of beam dynamics in polyharmonic field.

**MATRIX CALCULATIONS**

Conditions of longitudinal and transverse beam stabilities for the structure consisting from the periodic sequence of the cavities and solenoids were studied early using transfer matrix calculation [2]. The code window shows dependencies of dimensionless parameter \( \alpha = \frac{\pi eU L}{2 A \lambda m c^2} \) (for stable motion it must be less than 2), here \( U \) is accelerating potential per cavity, \( L \) is period length, phase advances per period (Floke parameters), magnetic field which is need for beam stable motion with envelope value \( X_m = 3 \text{ mm} \) is shown in Figure 2.

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\alpha = \frac{\pi eU L}{2 A \lambda m c^2} 
\]

Figure 2: Beam dynamic transfer matrix calculation.

**BEAM DYNAMICS IN SMOOTH APPROXIMATION**

In SC linac design, it is very important to know the bucket size since it relates to the longitudinal RF focusing. But the linac longitudinal acceptance cannot be obtained by matrix method because of the assumption that the particles have small longitudinal oscillation amplitude. In order to investigate the nonlinear ion beam dynamics in such accelerated structure and to calculate the longitudinal and transverse acceptances it can be used smooth approximation [3, 4].

We realize two smooth approximation methods in this program. In the first case take into account the coherent oscillations of bunches and the effective potential function describe slowly oscillations in the reference particle frame (Fig. 3a). In the second case the effective potential function was found in the frame where averaged velocity of the reference particle \( \beta_v = 0 \) (Fig. 3b) [5].

The analysis of the effective potential function makes it possible to study the condition of the beam phase and radial stability and to calculate the longitudinal acceptance.

The code window contain plots of longitudinal and transverse oscillations frequencies, the cross section of the potential function and change the size of the separatrix is shown in Figure 3.

**NUMERICAL SIMULATION OF ION BEAM DYNAMICS**

**Single Particle Motion**

The particle dynamic was studied in averaged on fast oscillations field for the analyses of longitudinal and transverse motions. Field’s components can be obtained from the effective potential function in smooth approximation. The solution can be obtained only by numerical simulation because the field components are nonlinear functions. The greatest interest has the modeling of the dynamics of the full field. The results of ion beam numerical simulations in polyharmonic field with \( Z/A = 1/66 \) are shown in Figure 4.

**Beam Dynamics**

As Initial parameters of numerical simulation for this system (see on figures) are similar to the previous one. The initial particle phase in the cavity are \( \phi_1 = -30^\circ \) and \( \phi_2 = 20^\circ \), and magnetic field of solenoid is \( B = 14 \text{ T} \). The beam moves through 8 periods in this case. The numerical simulation in polyharmonic field was performed to verify of the result obtained bellow. The geometrical velocity \( \beta_G \)
varies in each cavity. Results of ion beam numerical simulations in a polyharmonic field are close to results received in smooth approximation. The beam longitudinal size increase is negligible and transverse emittance slowly vying. The results are shown in Figure 5 agree with previously analyze.

**Figure 4:** Particle trajectories during acceleration.

**Figure 5:** Beam dynamics simulation for 1st smooth approximation.

**Figure 6:** Beam dynamics in the polyharmonic field.

**NUMERICAL SIMULATION OF ION BEAM DYNAMICS**

The numerical simulation in polyharmonic field was performed to verify of the result obtained bellow. The simulation was spent for the same focusing periods and the same initial parameters. Geometrical velocity $\beta_G$ varies in each cavity too. The results of ion beam numerical simulations in a polyharmonic field are close to results received in smooth approximation. The beam longitudinal volume increase negligible and transverse emittance slowly vying.

**CONCLUSION**

New BEAMDULAC-SCL code, developed for the beam dynamics analysis and accelerator parameters optimization (for linacs as SNS, FRIB etc) was described. The program window contains several tabs with different beam dynamics calculating methods: the matrix calculation, which is used to advance parameters choice of the accelerating structure, and nonlinear particle dynamics analysis in a smooth approximation, the calculation of the beam dynamics in the real field with slipping coefficient account to verify and refine the chosen accelerator parameters.

**REFERENCES**