

ALTERNATIVE APPROACHES FOR HOM-DAMPED CAVITIES*

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Abstract

In this paper, we present two different ideas that may be useful for design and simulation of (superconducting) radio frequency cavities.

To obtain longitudinal and transverse voltages resp. shunt impedances in cavities without rotational symmetry, one or two integration paths are often used to get an approximate difference relation for the transverse voltage of higher order modes (HOMs). The presented approach uses a multipole decomposition that is valid in vicinity of the central axis to compute voltage multipole decomposition directly for paths of arbitrary number and position.

Elliptical cavities have been a standard in SRF linac technology for 30 years. We present another approach to base cell geometry based on Bezier splines that is much more flexible in terms of optimization, while reaching equal performance levels.

POLAR PATH INTEGRAL ANALYSIS

In the following section, the longitudinal and transverse voltages are expressed using a multipole decomposition. By pseudo-inversion of the resulting equation system (similar to polynomial interpolation), one can calculate the corresponding coefficients directly from an arbitrary number and position of integration paths.

This method also allows to check a) if the position and number of paths is appropriate to extract the desired coefficients, b) if there is any “noise” in the voltage paths that stem from computational errors or from a too large distance of the path from the central axis.

Signed Effective Voltages

The effective longitudinal and transverse acceleration voltages can be written as complex Fourier integrals

$$\tilde{V}_{\parallel} = \int E_s(s) \exp(ik_{\beta}s) ds \in \mathbb{C}, \quad (1)$$

$$\tilde{\vec{V}}_{\perp} = \frac{1}{q} \int \vec{F}_{\perp}(s) \exp(ik_{\beta}s) ds \in \mathbb{C}^2, \quad (2)$$

where \vec{F}_{\perp} is the total transverse Lorentz force, and E_s is the longitudinal electric field at the path position s . The complex angle $\Psi_0 = \arg(\tilde{V}_{\parallel})$ corresponds to an optimal acceleration phase, and its magnitude to the acceleration

voltage for this phase (see [6] for a special derivation of this relation with a fixed $\Psi_0 = 0$).

For each mode, the phases of \tilde{V}_{\parallel} on all paths are equal up to an additional shift by π , corresponding to a sign change (deceleration). Using a fixed phase Ψ_0 that is one of the two possible optimal phases, we may define a real acceleration voltage for the optimal phase by

$$V_{\parallel} := \tilde{V}_{\parallel} \cdot e^{-i\Psi_0} \in \mathbb{R}. \quad (3)$$

and, using the Panofsky-Wenzel theorem, a corresponding real transverse voltage may be defined by

$$\frac{\partial}{\partial t} \tilde{\vec{V}}_{\perp} = i\omega \tilde{\vec{V}}_{\perp} = -\vec{\nabla}_{\perp} \tilde{V}_{\parallel} \quad (4)$$

$$\vec{V}_{\perp} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} = -i \tilde{\vec{V}}_{\perp} \cdot e^{-i\Psi_0} \in \mathbb{R}^2. \quad (5)$$

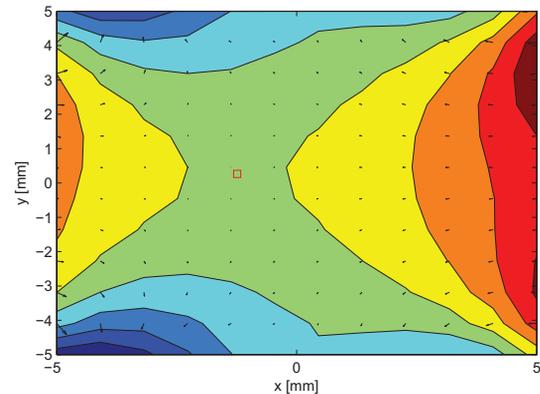


Figure 1: Reconstruction of the transverse (arrows) and longitudinal (isolines) voltages in the beam area with contributions from different multipole components. The corresponding coefficients were computed from voltages on circularly arranged integration paths at 5mm offset from beam axis for an eigenmode of the BERLinPro main linac cavity. The red square marks the transverse forceless point z_0 which deviates from the axis by more than 1 mm.

Holomorphic Multipole Expansion

By using a complex analytic multipole decomposition

$$F(z) \approx \sum_p c_p z^p, \quad (6)$$

where $z = x+iy = re^{i\phi}$ is the transverse position of the integration path, and $c_p = a_p + ib_p$ are complex numbers that

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describe the magnitude and orientation of different multipoles, the signed longitudinal and signed transverse voltages can be expressed as

$$V_{\parallel}(z) := \Im F(z) = \sum_p r^p [b_p \cos(p\phi) + a_p \sin(p\phi)] \quad (7)$$

$$\Rightarrow X(z) := \frac{d}{dz} F = -\omega(V_y + iV_x) = \sum_p p c_p z^{p-1} \quad (8)$$

Multiple Path Equation System

Assuming there are N numerical integration paths at transverse positions z_n set up for the computation, they can be combined into a vector thus summarizing the problem into two matrix equations.

$$\vec{V}_{\parallel} = \Im \left[\mathbf{M} \begin{pmatrix} c_0 \\ \vdots \\ c_{P-1} \end{pmatrix} \right], \quad \vec{X} = \mathbf{M} \begin{pmatrix} 1c_1 \\ \vdots \\ Pc_P \end{pmatrix} \quad (9)$$

where \mathbf{M} denotes the $N \times P$ Vandermonde matrix defined by $M_{n,p} = z_n^{p-1}$.

The matrix equation for \vec{V}_{\parallel} is sufficient to solve for the coefficients. On the other hand, the possible occurrence of numerical errors in the path integral computation and the limited validity of the multipole decomposition, which only holds for small $|z|$, enforces cross-checking with the transverse data. To get the best results, information of all V should be used simultaneously. To do this, rectangular matrices \mathbf{G} are defined by

$$\begin{pmatrix} c_0 \\ \vdots \\ c_{P-1} \end{pmatrix} = \mathbf{G}_1 \vec{c}, \quad \begin{pmatrix} 1c_1 \\ \vdots \\ Pc_P \end{pmatrix} = \mathbf{G}_2 \vec{c}, \quad (10)$$

where \vec{c} is a $P + 1$ -dimensional vector containing all c_p from 0 to P . The imaginary part of the bracketed term in (9) can be expressed in terms of the real and imaginary components \vec{a}, \vec{b} . One must now also separate \vec{X} into its real and imaginary multipole components. Furthermore, the resulting matrix equation is still complex, since \vec{X} is a complex vector, but \vec{V}_{\parallel} must be real (which leads to inversion problems). To resolve this, the final equation system can be written as

$$\begin{pmatrix} \vec{V}_{\parallel} \\ \Re \vec{X} \\ \Im \vec{X} \end{pmatrix} = \begin{pmatrix} \Im \mathbf{M} \mathbf{G}_1 & \Re \mathbf{M} \mathbf{G}_1 \\ \Re \mathbf{M} \mathbf{G}_2 & -\Im \mathbf{M} \mathbf{G}_2 \\ \Im \mathbf{M} \mathbf{G}_2 & \Re \mathbf{M} \mathbf{G}_2 \end{pmatrix} \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix}. \quad (11)$$

To solve this equation system for \vec{a}, \vec{b} , singular value decomposition

$$\begin{pmatrix} \vec{V}_{\parallel} \\ \vec{X} \end{pmatrix} := \mathbf{u} \Sigma \mathbf{v}^* \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix} \Rightarrow \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix} = \mathbf{v} \Sigma^{-1} \mathbf{u}^* \begin{pmatrix} \vec{V}_{\parallel} \\ \vec{X} \end{pmatrix} \quad (12)$$

is used. For voltages of a number of eigenmodes Q to be processed, their column $2N$ vectors can be concatenated to form a $2N \times Q$ matrix.

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3A Superconducting RF

Since multipole expansion of the longitudinal voltage is used, there must be at least one extremal point if components higher than dipolar are included. In fact, if the field is dominated by quadrupolar modes, one is interested in the exact position of this extremal point z_0 , since the transverse voltages included in $X(z)$ cancel out for this path. The extremal point z_0 is given by

$$X(z_0) = \sum_p p c_p z_0^{p-1} \stackrel{!}{=} 0, \quad (13)$$

which can easily be solved numerically for z_0 by finding the roots of the given polynomial.

To check if the number and position of paths for the given multipole order is sufficient, one should check for the ratio σ_1/σ_P to be not too large. To check if the multipole decomposition method is valid for the integration path region, one should investigate if the components larger than P of the vector $\mathbf{u}^* \begin{pmatrix} \vec{V}_{\parallel} \\ \vec{X} \end{pmatrix}$ do vanish.

Preliminary Results and Outlook

Figure 1 shows a reconstructed area around the central axis path for the BERLinPro main linac cavity for a mainly quadrupolar mode, and the forceless path position. The irregular isolines at the outer edge indicate a mixing with higher order multipole field components. For more results on the BERLinPro main linac, see [1].

The obtained information is important in the consideration of coupler kicks that do not stem from the operational mode, but from quadrupolar or higher order modes (HOM coupler kicks). The question whether to include mainly quadrupolar modes into HOM damping considerations may be answered using such studies, especially the z_0 position distribution along all HOMs of the cavity. Such a study may also help to answer questions regarding alignment tolerances of cavities and cavity strings.

For including more distant points from the axis into the equation, one may consider to relax the multipole conditions and to include Bessel function terms into the modal decomposition basis, thus allowing for two or more different complex coefficients to describe each multipole. As an extreme case for all points of a cylindrical mesh, this would be equivalent to a waveguide mode-like decomposition of the generalized path voltages on an electric boundary plane with $V_{\parallel}, iV_{\perp}$ instead of $\vec{E}_{\parallel}, i\vec{H}_{\perp}$.

BEZIER SPLINE CAVITIES

In modern accelerator facilities, high acceleration gradients and duty cycles are achieved using SRF cavities. To protect these cavities from quenching and RF breakdown, sufficiently small peak surface electric fields in comparison to acceleration gradient $E_{\text{surf}}/E_{\text{acc}}$ are necessary. Therefore, the cavity shapes must be constructed from smooth profile curves.

From the disk-loaded cavity, design of such cavities has evolved from first rounding off the edges, and then substituting them by elliptical shapes, leading to modern SRF "re-entrant", "low-loss" and "TESLA-shape" [5] cavities which suppress resonant multipacting of secondary electrons. While elliptical shapes have shown superior performance, this parameterization is qualified by historical developments, as to the authors' knowledge, no other parameterizations of similar smoothness have been systematically researched for ultra-relativistic particles. The spline cavity geometry is such an alternative shape with additional desirable properties [2].

Figure of Rotation Parameters

A Bezier spline is a parameterized curve [3]

$$\vec{s}(t) = \vec{a}_0 + \vec{a}_1 t + \vec{a}_2 t^2 + \dots = \sum_{n=0}^N \vec{s}_n b_{n,N}(t) \quad (14)$$

with Bernstein polynomials $b_{n,N}$ and $t \in [0, 1]$. The $N + 1$ points \vec{s}_n define the so-called control polygon and contain the free parameters of the spline. Let us consider the simplest possible case of a cubic ($N = 3$) Bezier spline as shown in Fig. 2. The geometry parameters are reduced by cell periodicity conditions and elementary RF constraints to a set of only three parameters [2], compared to five for an elliptical cavity. These parameters can be optimized according to the specific purpose of the cavity.

Implementation and Results

For the single cell computations with periodic boundary conditions, we used the well-known 2d code SUPERFISH [4]. Since the SUPERFISH mesher cannot use spline curves, a small wrapper for MATLAB was developed. The spline geometry was discretized by calculating a 200 point (halfcell) polygon from its parameters (Fig. 2), which was then used as input in SUPERFISH, also specifying a special localized mesh that matches the polygon discretization accuracy.

A parameter sweep for a given iris radius and an external equator tuning procedure were also implemented to tune all cavity sets to a π -mode frequency of 1.3 GHz [2]. The scan of the $(v_1, v_2)/L$ parameter space was processed in a spiral pattern to fully exploit the similarities between spline shapes with only slight parameter differences. A second scan with given radii for the 0-mode was performed, obtaining the corresponding frequency needed for the intercell coupling constant κ [6]. The procedure for a fixed iris radius took approx. 3.5 hours on a standard PC [2].

Figure 3 shows the results for a typical iris radius of 35 mm. It can be seen by comparison e.g. with [5] that the cubic spline shape geometry performance is in roughly the same range as the standard elliptical cavity geometries, although as stated, the cubic shape has two less free parameters. By inspecting the map, the trade-off between differ-

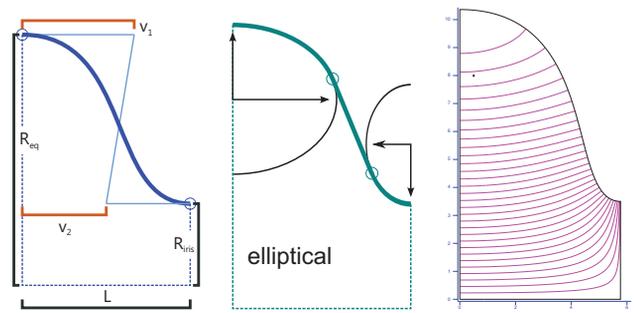


Figure 2: Parameterization for cubic Bezier shape (left), elliptical parameterization (center) and exemplary result for electric field lines ($v_1 = 0.9, v_2 = 0.65, R_{iris} = 35$ mm, $R_{eq} = 103.60$ mm, right)

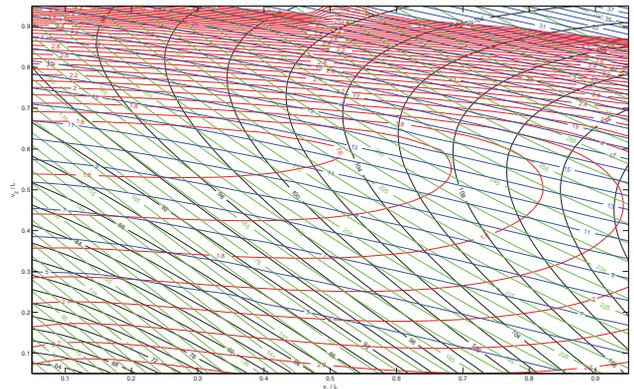


Figure 3: Cubic Bezier map for a 1.3 GHz cavity with iris radius 35 mm. The map shows the cubic interpolated iso-lines of R/Q [Ω] in black, E_{surf}/E_{acc} in red, the Geometry factor [Ω] in green, and the intercell coupling constant [10^{-3}] in blue.

ent figures of merit in the design process becomes visible (see e.g. red and black lines).

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