LINAC OPTICS DESIGN FOR MULTI-TURN ERL LIGHT SOURCE*


Abstract

The optics simulation group at HZB is designing a multi-turn energy recovery linac-based light source. Using the superconducting Linac technology, the Femto-Science-Factory (FSF) will provide its users with ultrabright photon beams of angstrom wavelength at 6 GeV. The FSF is intended to be a multi-user facility and offer a variety of operation modes. In this paper a design of transverse optic of the beam motion in the Linacs is presented. An important point in the optics design was minimization of the beta-functions in the linac at all beam passes to suppress beam break-up (BBU) instability.

INTRODUCTION

In this document we present a design of a Linac optics for a new 3 pass ERL-based LS with 6 GeV maximum energy of electron beam. This future facility is named Femto-Science Factory (FSF) [1].

The schematic layout of the facility is presented in Fig. 1. A beam is created in a 1.3 GHz SRF gun with photo cathode. We consider an SRF injector with similar parameters to the BERLinPro injector under development at HZB [2, 3]. Then it passes a 100 MeV Linac and is accelerated to 6 GeV after passing 3 times through each of two 1 GeV main Linac’s. In the arcs between the acceleration stages it is assumed to have undulators with 1000 periods. In the long straight section (see Fig.1) a long undulator with 5000 periods is assumed. The main design parameters of FSF are presented in Table 1.

Figure 1: The scheme of FSF.

One potential weakness of the ERLs is transverse beam breakup instability, which may severely limit a beam current. If an electron bunch passes through an accelerating cavity it interacts with dipole modes (e.g. TM_{110}) in the cavity. First, it exchanges energy with the mode; second, it is deflected by the electro-magnetic field of the mode. After recirculation the deflected bunch interacts with the same mode in the cavity again which constitutes the feedback. If net energy transfer from the beam to the mode is larger than energy loss due to the mode damping the beam becomes unstable.

The actuality of this problem was recognized in early experiments with the recirculating SRF accelerators at Stanford [4] and Illinois [5], where threshold current of this instability was occurring at few microamperes of the average beam current. In the works of Rand and Smith in [6] dipole high order modes were identified as a driver of this instability. In late of the 80’s the detailed theoretical model and simulation programs had been developed [7, 8]. Nowadays the interest to this problem was renewed.

The requirements for more detailed theory and simulation programs [9-11] are given by the needs of high current (~100 mA) ERLs. The threshold current for the transverse beam breakup may be estimated for the case of a single cavity and single mode for a multipass ERL in the form as [11]:

\[ I_{th} \approx I_0 \frac{\lambda^2}{Q_a L_{eff} \sum_{m=1}^{2N-1} \sum_{n=m+1}^{2N} \beta_m \beta_n}, \]

where \( I_0 \) - Alfven current, \( Q_a \) is the quality factor of HOM, \( \lambda = \lambda_{cm} \), \( \lambda \) is the wavelength corresponding to the resonant frequency of the TM_{110} mode, \( n \) is the relativistic factor at the m-th pass through the cavity, \( \beta_m \) – is the Twiss parameter, \( L_{eff} \) – is the effective length of the cavity, \( N \) is the number of passes during acceleration. It should be noted that (1) gives more realistic estimation of the BBU threshold current than a similar expression in [9] with a 1/NI(2N-1) dependence on the number of passes. This is the result of an assumption in [9] of integer tunes in every turn of the ERL.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>High brilliance mode</th>
<th>Short bunch mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>E, GeV</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>&lt;I&gt;, mA</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Q, pC</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>&lt;t&gt;, fs</td>
<td>200-1000</td>
<td>~10</td>
</tr>
<tr>
<td>&lt;B&gt;, ph/s/mm2/mrad2/0.1%</td>
<td>8\times10^{22}</td>
<td>~4\times10^{23}</td>
</tr>
<tr>
<td>B_{peak}, ph/s/mm2/mrad2/0.1%</td>
<td>10^{26}</td>
<td>~10^{26}</td>
</tr>
</tbody>
</table>

Eq. 1 shows that it is preferable to have low \( \beta \)-functions at low energies. Therefore, the design was optimised to minimize beta functions of the beam in the Linac to increase the threshold current of BBU instability from one...
point of view and to decrease the beats of the beta functions in the spreaders from another.

As it was shown in [1], for a scheme of FSF, presented in Fig. 1, BBU instability will develop in the 1st cavity of the 1 GeV linac.

In this paper we analyze the influence of the energy gain in the preinjection linac (100 MeV) and two main linacs (1 GeV) on the BBU threshold and propose an optimized solution.

**Preinjection Linac Pattern**

In this part we would like to discuss BBU instability in the preinjection linac, which is coming after the merger section. In Fig. 1 it is suggested to have a 100 MeV linac. Therefore we have acceleration from 7 to 107 MeV, or deceleration for the same amount of energy. Therefore it is required to have one cryomodule with an acceleration gradient of the cavities of $G = 15.5$ MeV/m.

Now we would like to analyze model with a linac consisting of two cryomodules with acceleration up to 250 MeV and with a triplet of quadrupole magnets in between them, which role is to change the sign of the twiss parameter $\alpha$ of the beam. Let us find the initial injection twiss parameters to have the equal threshold currents for the entrance and for the middle of the linac. To find that we will use eq. 1 and a model of the linac with one dipole HOM, which we locate at different positions in the linac. In this case we assume that the 1st cryomodule is a long cavity with a transfer matrix $M$ given by [12]:

$$
\begin{pmatrix}
\cos(\alpha) - \sqrt{2} \sin(\alpha) \\
-\frac{3}{\sqrt{8}} \frac{\Delta \gamma \sin(\alpha)}{L \gamma_i} \\
\end{pmatrix}
\begin{pmatrix}
\sqrt{8} \gamma_0 L \sin(\alpha) \\
\Delta \gamma \\
\end{pmatrix},
\tag{2}
$$

where $\alpha = \frac{1}{\sqrt{8}} \ln \left( \frac{\gamma_f}{\gamma_0} \right)$, $\gamma_{f(0)}$ is the final(initial) normalized energy of the particle, $L$ – the length of the cavity (cryomodule). Also we assume that there is a symmetrical $\beta$-functions on acceleration and deceleration in the linac. Therefore we can transfer the beta function through the 1st cryomodule as:

$$
\beta_1 = \frac{\gamma_L}{\gamma_0} \left( \beta_0 m_{i1}^2 - 2\alpha_0 m_{i1} m_{i2} + \frac{1 + \alpha_0^2}{\beta_0} m_{i2}^2 \right),
\tag{3}
$$

The RF focusing in the second cryomodule is quite weak and we represent it with a cavity without RF focusing for simplicity. So the beta function at the end of the linac might be found as:

$$
\beta_2 = \frac{\gamma_2}{\gamma_1} \left( \beta_1 - 2\alpha_1 L + \frac{1 + \alpha_1^2}{\beta_1} L^2 \right),
\tag{4}
$$

The minimum of the $\beta_2 = 2L^2 / \beta_1$ is given, when

$$
\alpha_1 = \frac{\beta_1}{L}.
\tag{5}
$$

Now we can proceed with an equation for the thresholds:

$$
\sqrt{\frac{\beta_0 \beta_2}{\gamma_0 \gamma_2}} = \frac{2\beta_1}{\gamma_2 - \gamma_0},
\tag{6}
$$

To find the solution we will vary the initial twiss parameters and look for a maximum value of the thresholds. The numerical solution is presented in Fig. 2.

![Figure 2: Threshold current, calculated using eq. 1](image)

One can see from Fig. 2 that the maximum threshold current is achieved at $\alpha_0 \sim -1.26$ and $\beta_0 \sim 3.41$.

**Different Acceleration Pattern**

In this paragraph we would like to discuss an improvement of the present scheme of FSF, where we have 100 MeV preinjection and then two 1 GeV Linacs (Fig. 1), for the scheme, presented in Fig. 3.

![Figure 3: The improved acceleration scheme of FSF](image)
like to find a balance between the energy gains in two main linacs to have equal threshold currents for them. To do that, we will analyse a model with linacs, when a focusing from a triplets is neglected for the second and the third passes. In this model with the injection energy of about 250 MeV the transverse focusing inside the cavities can be neglected. So, we assume that the beta functions of a beam at the exit and at the entrance to the linac are about the length of the linac for the second and the third passes and for the end of the linacs at the first pass and it is about the length of the one cryomodule [1] at the entrance for the first pass.

Let’s introduce $G$ as a gradient of the cavities in MeV/m, $L=2000$ [MeV] /$G$ is a length of the cavity structure, required to accelerate to the final energy of 2 GeV, $x$ is the length of the first linac and, therefore, $L-x$ is the length of the 2-nd. Now we can find energies $\gamma_{1(2),n}$ for each pass and as we said before $\beta_{1,1(n)} = \beta_{2,1(n)} = 12.57$ m and $\beta_{1,n} = x$ or $\beta_{2,n} = L-x$ for the first and the second linac respectively and for $n=2..5$.

We can proceed with the following equation:

$$
\sum_{n=1}^{N} \sum_{m=1}^{N} \frac{\beta_{1,n} \beta_{1,m}}{\gamma_{1,n}(x) \gamma_{1,m}(x)} = \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{\beta_{2,n} \beta_{2,m}}{\gamma_{2,n}(x) \gamma_{2,m}(x)}, \quad (7)
$$

when the threshold currents have the same values for the 1-st and last cavities in both linacs. This equation can be solved numerically and gives the result that $x \sim L/3$ with injection energy $\gamma_{1,1} = 480$. This explains why we propose a new acceleration pattern in Fig. 3.

Now we proceed with a modeling of the linac optics in Elegant program [13]. Optics for all three passes through the first 666 MeV linac is presented in Fig. 4.

Figure 4: Optics design of the first 666 MeV linac. 3 passes with 250, 2.250 and 4.250 GeV beam injection energy from left to right correspondingly.

Also the optics was designed for the second 1334 MeV linac and it is presented in Fig. 5.

Figure 5: Optics design of the second 1334 MeV linac. 3 passes with 916, 2916 and 4916 MeV beam injection energy from left to right correspondingly.

The threshold currents for the schemes of FSF in Figs. 1 and 3 are presented in Table 2. For the estimations we used the following equation (which is a combination of eq. 1 and [9]):

$$
I_{th} = \frac{2mc^3}{e\alpha}\left(\frac{R}{Q}\right)\frac{Q_d}{d} \sqrt{\sum_{m=1}^{5} \sum_{n=m+1}^{6} \frac{\beta_m \beta_n}{\gamma_m \gamma_n}}, \quad (8)
$$

and we took a mode with $(R/Q)_{d}Q_d=6 \cdot 10^{5}$ Ω, $\omega = 2 \pi \cdot 2 \cdot 10^{12}$ Hz.

### Table 2: Threshold Currents for Different Schemes

<table>
<thead>
<tr>
<th>Linac scheme</th>
<th>$I_{th}, A$</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; Linac</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; Linac</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preinjector</td>
<td>1.26</td>
<td>0.88</td>
<td>3.73</td>
</tr>
<tr>
<td>0.100 + 1 + 1GeV</td>
<td>1.21</td>
<td>1.46</td>
<td>3.58</td>
</tr>
<tr>
<td>250 + 666 + 1334MeV</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One can see from the Table 2 that the value of threshold current for the 1<sup>st</sup> main linac was improved. It was also slightly decreased for the 2<sup>nd</sup> linac and for the preinjection.

### CONCLUSION

In the new proposed scheme BBU instability will develop in the preinjection Linac. In the future work we should find a way how to increase the thresholds in the preinjection linac and in the 1<sup>st</sup> by decreasing the threshold current value in the 2<sup>nd</sup> linac.

### REFERENCES