

LATTICE MATCHING WITH ELEGANT AT ELSA

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Abstract

The electron stretcher ring ELSA provides a beam of polarized electrons of up to 3.2 GeV energy. To preserve the initial degree of polarization, several depolarizing resonances have to be compensated during the fast energy ramp of 6 GeV/s. Beam depolarization, caused by crossing these resonances is studied using comprehensive numerical calculations. These depend essentially on a precise model of the actual magnetic field distributions, explicitly taking into account misalignments. Hence it is necessary to match the theoretical lattice to the actual accelerator. In a first step the alignment of all magnets has been examined and improved. This was done by using standard survey equipment and precise electronic spirit levels. In a second step the concept of response matrix fitting is used for further, beam based, lattice matching. Particle tracking and optics calculations are carried out using *ELEGANT*, a fully 6D accelerator toolbox. Lattice matching is done by repeatedly calling *ELEGANT* and utilizing a Levenberg-Marquardt optimizer. In this contribution we will describe our lattice fitting implementation.

INTRODUCTION

ELSA is a 164 m long separated function machine with FODO-structure (see Fig. 1). It consists of 24 bending dipole magnets and 32 quadrupole magnets, divided in two families. For chromaticity correction and for excitation of a third integer betatron resonance needed for slow beam extraction, twelve sextupole magnets, divided into three families, are installed.

At ELSA the so called *SIMLIB*, an in house developed simulator, is used as an online model. Up to now this model does not consider misalignments and field errors of individual magnets. This leads to substantial differences between the model and the real accelerator. For example up to now empirical calibration factors between quadrupole strength and desired tunes have to be used. Also *SIMLIB* cannot provide field maps of individual magnets and is not able to take edge effects into account. Therefore it cannot be used as input for numerical simulations of beam depolarization [1].

As a successor of *SIMLIB* currently the computer code *ELEGANT* [2] is introduced at ELSA. In order to bring the model in good agreement with the real accelerator a beam based approach is implemented. Therefore the model is optimized to match the simulated orbit response matrix to the measured matrix. This is done in a similar way as described in [3], the so called LOCO (Linear Optics from Closed Orbits) method.

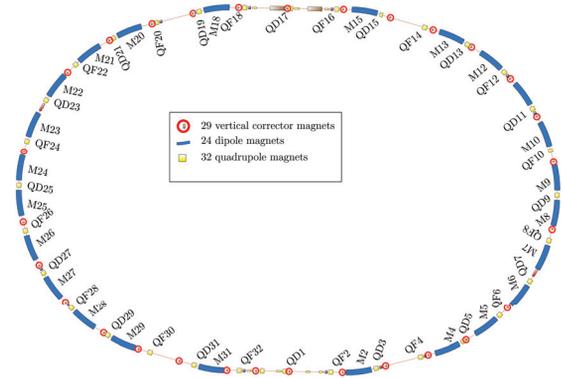


Figure 1: Sketch of the electron stretcher facility ELSA.

ORBIT RESPONSE MATRIX

The orbit response matrix (ORM) describes the response of the closed orbit \vec{u} to the change of applied corrector magnet kicks $\vec{\alpha}$:

$$\vec{u} = O \vec{\alpha} \quad O^{M \times N} = (\vec{O}_1, \dots, \vec{O}_N)$$

Element O_{ik} of the orbit response matrix represents the orbit shift at the i -th beam position monitor due to a kick from the k -th corrector magnet:

$$O_{ik} = \frac{\partial u_i}{\partial \alpha_k} \quad (1)$$

In linear approximation the matrix O does not depend on the kick strength $\Delta\alpha$.

At ELSA a beam position monitor (BPM) is installed at every quadrupole magnet, leading to 32 measurable beam positions in each plane. For closed orbit correction in the horizontal plane extra trim windings of the 24 main dipole magnets are used, in the vertical plane 29 dedicated corrector magnets are available. This leads to $(32 + 32) * (24 + 29) = 3392$ entries of the ORM.

For a detailed description of the closed orbit correction setup see [4].

ORM Measurement

In order to measure column k of the orbit response matrix the kick of the k -th corrector is changed by a small amount $\Delta\alpha$ and the resulting closed orbit $\vec{u}(\Delta\alpha)$ is measured. The kick is changed to $-\Delta\alpha$ and the resulting closed orbit is measured again. The difference between the two measured closed orbits:

$$\Delta\vec{u} = \vec{u}(\Delta\alpha) - \vec{u}(-\Delta\alpha)$$

divided by the kick difference leads to the orbit response:

$$\vec{O}_k \approx \frac{\vec{u}(\Delta\alpha) - \vec{u}(-\Delta\alpha)}{2\Delta\alpha} \quad (2)$$

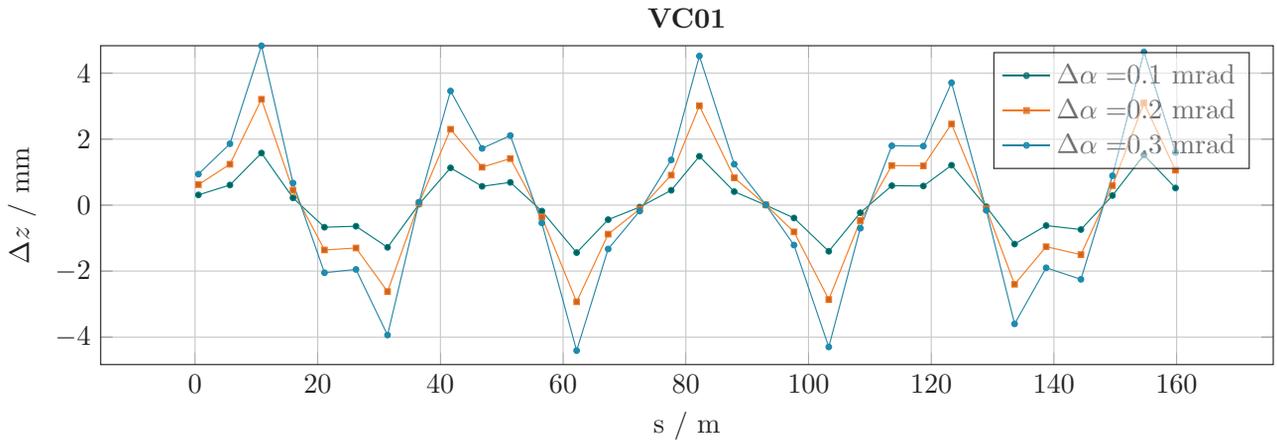


Figure 2: Shown are the measured closed orbit differences $\Delta z = z(\Delta\alpha) - z(-\Delta\alpha)$ for three different corrector kicks of vertical corrector magnet VC01.

To assure that the linear approximation still holds the measurement is repeated for different values of $\Delta\alpha$ (see Fig. 2). For every entry of \vec{O}_k the measured orbit differences Δu_i are then fitted against $\Delta\alpha$ and the corresponding response entry is deduced.

ORM Simulation

In principle the model ORM can be calculated by using the simulated beta and phase functions, β and ψ , at all beam positions monitors and corrector magnets [5, sec. 4.7.3]:

$$O_{ik} = \frac{\sqrt{\beta_i \beta_k}}{2 \sin \pi Q} \cos (|\psi_i - \psi_k| - \pi Q) \quad (3)$$

Here Q is the corresponding betatron tune. At ELSA the horizontal correction is implemented by changing the dipole field via extra trim windings of the main bending magnets.¹ Because the dipole magnets are almost 3 m long the beta and phase functions cannot be approximated as constant along the longitudinal direction of the magnet and therefore Eq. (3) cannot be used.

Fortunately ELEGANT can be configured to use tracking techniques to find the orbit response. The columns of the ORM are calculated similar to the measurement procedure: After modifying the kick of the k -th corrector magnet by a small amount $\Delta\alpha$ a new closed orbit is calculated by tracking a test particle. Afterwards the corrector kick is changed to $-\Delta\alpha$ and a new closed orbit is tracked. Column k of the simulated ORM can then be calculated with Eq. (2).

MATCHING TECHNIQUE

In order to match simulated and measured ORMs a set of lattice parameters \vec{P} is adjusted to minimize the difference between the two matrices. The vector \vec{P} represents a set of parameters, such as gradient errors and displacements of

¹ the tracking code of ELEGANT was slightly modified in order to use the bending dipoles as corrector magnets. The bending field is adjusted according to the assigned corrector kick.

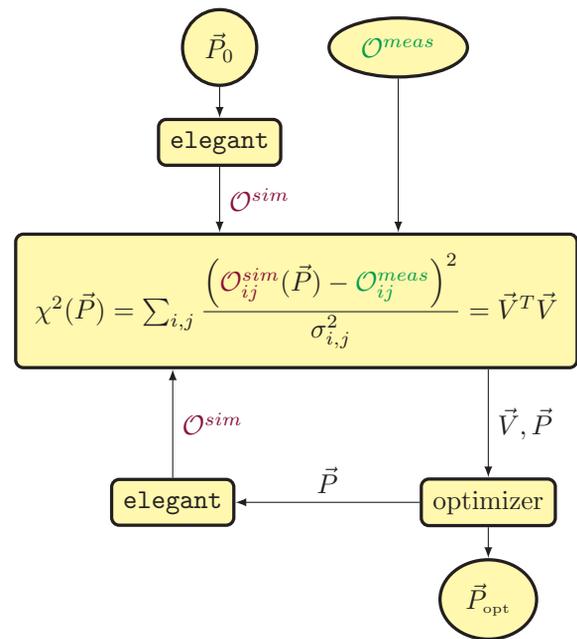


Figure 3: Schematic overview of the implemented fitting scheme.

quadrupole magnets, fractional field errors of dipole magnets, gains of BPMs and calibration factors for corrector magnets.

As the objective for an optimizer the following least-square problem can be formulated:

$$\chi^2(\vec{P}) = \sum_{i,k} \frac{(O_{ik}^{sim}(\vec{P}) - O_{ik}^{meas})^2}{\sigma_{i,k}^2} = \vec{V}^T \vec{V} \quad (4)$$

The minimization is implemented in a c++ program described in Fig. 3. Starting from an initial set of parameters $\vec{P} = \vec{P}_0$, a modified lattice is build and O^{sim} is calculated by ELEGANT. In a next step $\chi^2(\vec{P})$ is computed using Eq. (4) and O^{meas} as well as its measurement errors σ . Simultaneously the vector \vec{V} is constructed according to Eq. (4).

To find the solution for this nonlinear least-square problem a Levenberg-Marquardt optimizer [6, sec. 15.5], using the implementation from ALGLIB [7], is applied. This algorithm incrementally decreases $|\vec{V}|$. Therefore the Jacobian matrix

$$J_{ij} = \frac{\partial V_i}{\partial P_j} \quad (5)$$

has to be evaluated repeatedly to drive \vec{P} towards the minimum of χ^2 . After each step of the algorithm a new set of parameters \vec{P} is generated. These are then fed to ELEGANT in order to calculate a new O^{sim} . If the optimization converged to a minimum a set of optimal parameters \vec{P}_{opt} is found.

The most time consuming task is the calculation of O^{sim} . Almost 100 % of the overall computation time, usually one to two seconds on a desktop PC, is spent in calling ELEGANT, computing the ORM.

Calculation of Jacobian Matrix

Because the vector \vec{V} in Eq. (5) cannot be calculated analytically, the Jacobian matrix has to be found by numerical differentiation. Because several orbit response calculations via ELEGANT have to be performed it is crucial to optimize this step. Fortunately this can be done in parallel, because the columns of the matrix are independent of each other. For a small value ϵ the j -th column of J can be approximated as

$$\vec{J}_j \Big|_{\vec{P}_0} = \frac{\partial \vec{V}(\vec{P}_0)}{\partial P_j} \approx \frac{\vec{V}(\vec{P}_0 + \vec{\epsilon}) - \vec{V}(\vec{P}_0 - \vec{\epsilon})}{2\epsilon} \quad (6)$$

with $\vec{\epsilon} = \vec{e}_i \epsilon$. An ideal value for ϵ is not easy to find. It heavily depends on the scale of P_i and the accuracy of the simulated ORMs. The solution for this problem is to use an adaptive algorithm similar to the implementation in [6, sec. 5.7]. It calculates the derivative by iteratively reducing ϵ in Eq. (6). If the estimated error does not further improve the algorithm terminates. Usually the algorithm converges after a few iterations and does not depend on the initial ϵ .

SCALE

The Levenberg-Marquardt algorithm is usually quite robust concerning the scale of the optimized parameters. However if the parameter scale differs in several orders of magnitude it is necessary to pre-scale the problem, hence preliminary simulations are required. This can be done independently for each parameter type.

A random vector \vec{P}_{test} of normal distributed values is generated and multiplied with c . Then $\chi^2(c \vec{P}_{\text{test}})$ is simulated via ELEGANT (see Eq. (4)) for several orders of magnitude of c , and a polynomial fit of second order is performed.

In order to deduce the proper scaling for every parameter type the found polynomial is solved for an arbitrary chosen

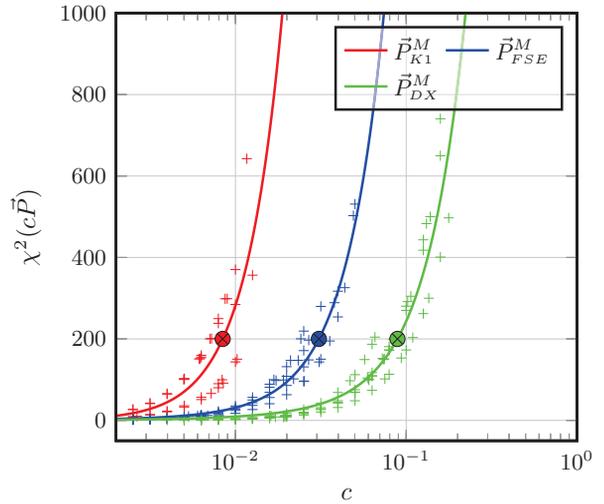


Figure 4: Simulated scale of quadrupole field components \vec{P}_{K1}^M , dipole field errors \vec{P}_{FSE}^M and horizontal displacements \vec{P}_{DX}^M of all 24 dipole magnets.

value of $\chi^2(c\vec{P}) = 200$ (see Fig. 4). The found c then can be used as the scaling factor.

Proper scaling is also necessary to prevent the χ^2 calculation from escalating into high values. If only a few parameters are badly scaled, tracking and hence the generation of ORMs can fail completely.

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