

# STUDIES ON DEPOLARIZATION BY SYNCHROTRON RADIATION USING ELEGANT PARTICLE TRACKING \*

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## Abstract

Spin dynamics in circular electron accelerators are significantly influenced by the emission of synchrotron light. In storage rings, Sokolov-Ternov polarization build-up and radiative depolarization have crucial impact on equilibrium polarization. On shorter timescales, as in damping rings or synchrotrons with fast energy ramp, the temporal development of polarization depends on spin decoherence caused by stochastic momentum changes. Thus, especially longitudinal beam dynamics affect depolarization. This contribution presents the implementation of particle tracking with synchrotron radiation from ELEGANT in an in-house developed spin tracking code. Exemplary results on depolarization including synchrotron radiation are shown.

## INTRODUCTION

Spin dynamics in circular electron accelerators are significantly influenced by the emission of synchrotron light, but the relevant effects severely depend on the operation timescale. Storage rings benefit from the Sokolov-Ternov effect slowly polarizing the beam due to spin flips during the emission of synchrotron radiation photons. Here, the main interest is equilibrium polarization along a stable spin axis, which results from polarization build-up and radiative depolarization. This equilibrium polarization can be calculated based on the Derbenev-Kondratenko formula.

This contribution deals with shorter timescales below the polarization build-up time, as in damping rings or synchrotrons with energy ramp, where an already polarized beam is injected. Here, also polarization components perpendicular to the stable spin axis are relevant and various time-dependent effects account for studying depolarization with spin tracking codes based on 3D spin precession according to the Thomas-BMT equation. This requires particle tracking, because both synchrotron and betatron motion affect spin dynamics. Thereby, synchrotron radiation has to be taken into account, but many spin tracking codes do not cover synchrotron radiation, as they are commonly used for proton beams.

Currently, a new spin tracking code is developed in-house at ELSA aiming for a realistic synchrotron radiation model. This contribution presents an approach using the 6D particle tracking from ELEGANT including synchrotron radiation as basis for the spin tracking. That is, the particle trajectories are recorded during ELEGANT execution and then imported by the spin tracking code. The implementation is described, as well as first results of the spin tracking with synchrotron radiation.

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## SPIN TRACKING IMPLEMENTATION

The spin tracking is performed with an in-house developed C++ code to allow flexible testing of particle tracking and synchrotron radiation implementations. The library LIBPALATTICE is used as interface to lattice information. It is also developed at ELSA and described in the next section. Though, the spin tracking algorithm can be changed with little effort.

First, the spin tracking program POLE [1] was used, which applies a Runge-Kutta integration of the Thomas-BMT equation with adaptive step size. It was especially developed to efficiently simulate the crossing of integer depolarizing resonances. For that purpose tracking accuracy could be balanced against computing time by frequency filtering of the magnetic fields. However, disadvantages of this field parametrization have been identified [2] and it was replaced by an element by element matrix tracking.

This spin tracking code, called POLEMATRIX, solves the Thomas-BMT equation as

$$\vec{S}(s_0 + l) \approx \mathbf{R}_{3 \times 3}(s_0, l) \vec{S}(s_0), \quad (1)$$

where the spin vector  $\vec{S}$  is tracked from the position  $s_0$  through a magnet with length  $l$  by a three dimensional rotation matrix  $\mathbf{R}_{3 \times 3}$ . The matrix contains the spin precession in the magnet and is constructed from the corresponding energy normalized magnetic field  $\vec{B} := e\vec{B}/(\beta\gamma mc)$ . Its direction determines the precession axis and the rotation angle  $\theta$  is calculated from the absolute value according to the Thomas-BMT equation:

$$\theta = \xi_{\vec{z}} \cdot \left| \vec{B}(x, z) \right| \cdot l \quad (2)$$

with transverse  $\xi_x = \xi_z = \gamma a$  corresponding to the spin tune and longitudinal  $\xi_s = a$ . Here,  $a$  is the gyromagnetic anomaly and  $\gamma$  the Lorentz factor. The linear algebra calculations are performed using the C++ library ARMADILLO. Multiple spins can be tracked in parallel with an adjustable number of threads. Afterwards, the polarization vector is calculated as the average over all spin vectors for each output step. The program options are set in an xml configuration file.

Synchrotron motion and energy ramps are exclusively contained in  $\gamma$  as a function of time for each particle. Transverse beam dynamics are included in the magnetic field  $\vec{B}(x, z)$ , which depends on the horizontal ( $x$ ) and vertical ( $z$ ) particle trajectory. Several models for both longitudinal and transverse dynamics can be selected in the configuration file. One option is the import from ELEGANT.

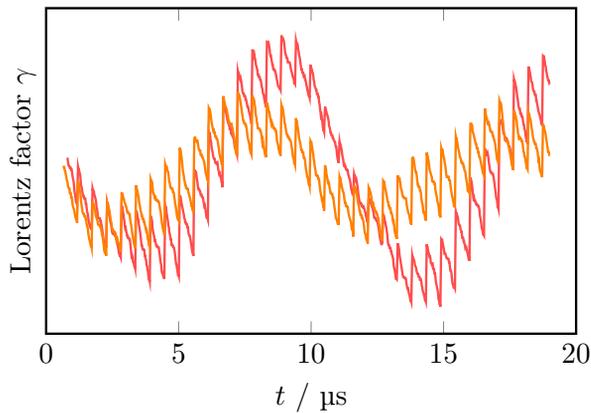


Figure 1: Section of exemplary ELEGANT output of the energy of two electrons emitting synchrotron radiation.

### IMPORT OF ELEGANT PARTICLE TRACKING DATA

ELEGANT is an established 6D particle tracking code actively developed at the Advanced Photon Source [3]. It includes synchrotron radiation, as well as several options to set up RF cavities and parallelization of tracking multiple particles [4]. All ELEGANT output is written to compressed binary so-called SDDS files. This allows to store detailed particle tracking data even for long tracking times. SDDS interfaces are available to command line, Java, Python and Matlab. Also the C API was recently made accessible (*libSDDS1* from *SDDSToolKit-devel* package). For these reasons, ELEGANT was chosen. Figure 1 exemplifies the synchrotron motion computed by ELEGANT consisting of stochastic energy loss in bending magnets and energy gain in the cavities together forming the synchrotron oscillation.

The SDDS C API is used to import the horizontal and vertical position ( $x, z$ ) and energy  $\gamma$  of a particle into a C++ data structure, which then allows access to any position via spline interpolation – e.g. to be used in Eq. 2. Import and access are thread safe. This data structure is part of the C++ particle accelerator lattice library LIBPALATTICE developed at ELSA and can be used to import any output of ELEGANT or MAD-X into C++ programs. The tracking is executed automatically if needed. Basic parameters, like energy or number of turns, can be set directly in C++. Furthermore, LIBPALATTICE provides a convenient data structure to access and edit lattices with all element properties, which can also be automatically imported from and exported to ELEGANT or MAD-X. So, a lattice file conversion tool is already included. Additionally, lattices can be exported to  $\LaTeX$  to sketch an accelerator with the TIKZ-PALATTICE package [5]. LIBPALATTICE will be published as open source software on GitHub soon.

To be able to readout betatron and synchrotron motion, ELEGANT is configured to save the 6D phase space coordinates of each particle in every quadrupole during tracking (WATCH files). When tracking 25 ms this leads to about 100 MB data per particle. Accessing them during spin track-

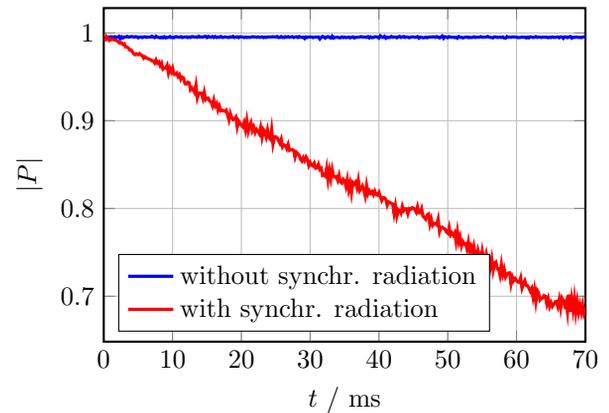


Figure 2: Absolute value of Polarization for 100 particles with 3.2 GeV starting all aligned in longitudinal direction.

ing takes a noticeable part of computing time. Therefore, reasonable usage of this approach on common PC hardware ends in the order of 100 ms tracking with 100 particles. This is sufficient to study the influence of synchrotron radiation and serve as benchmark for other particle tracking implementations in POLEMATRIX.

### SPIN DECOHERENCE

One consequence of synchrotron motion is a possible decoherence of the individual spin precessions, because the precession frequency, expressed as spin tune  $Q_{\text{spin}} = \gamma a$ , is proportional to the particles energy. The energy spread of a beam leads to a spin tune spread. This determines the temporal development of polarization if the spin vectors are not aligned parallel to the stable spin axis – e.g. after crossing a depolarizing resonance. In a flat circular accelerator (without solenoids) the stable spin axis usually points in vertical direction due to the strong guiding fields of the bending magnets. So, decoherence causes fading away of horizontal and longitudinal polarization.

A discussion of decoherence by deterministic forms of energy spread can be found in [6, chapter 21]. Obviously, a temporally constant energy spread causes a constant precession velocity spread and complete spin decoherence. Averaging over all spin vectors, only the component parallel to the precession axis remains: The beam is depolarized. The depolarization time depends on the energy spread. But this is not a realistic model for a radiating electron beam, where all particles oscillate around the same reference energy. The electrons perform synchrotron oscillations. These periodic energy modulations average to zero and can only cause spin decoherence if their period is long enough compared to a spin precession. However, if these oscillations are deterministic, the spins recohore periodically. Synchrotron radiation adds a stochastic part to the spin precession frequency and thus drives irreversible spin decoherence and depolarization of an electron beam.

To test if this can be observed using the synchrotron radiation model from ELEGANT, a spin tracking with spins starting

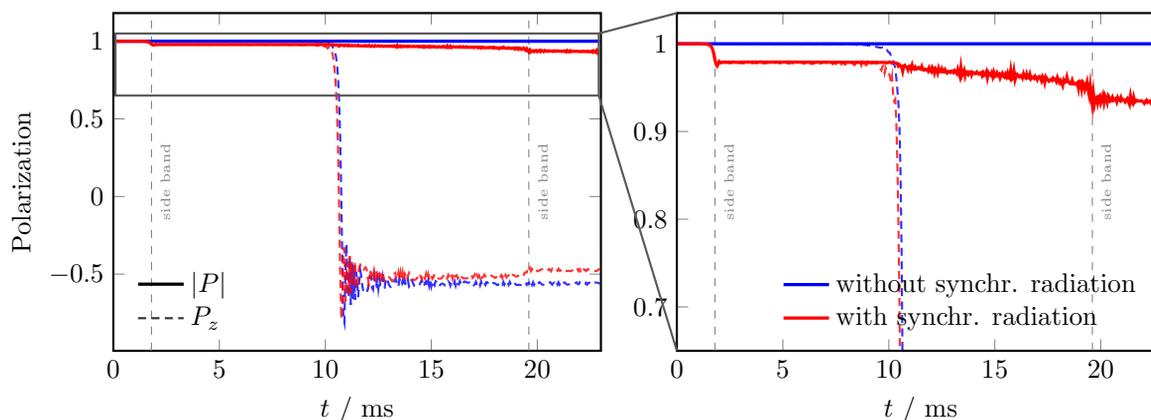


Figure 3: Crossing an integer resonance with 4 GeV/s with 100 particles starting in vertical direction.

perpendicular to the stable precession axis is used. Figure 2 shows an exemplary spin tracking result from POLEMATRIX for the ELSA stretcher ring at 3.2 GeV. The spins of 100 particles start aligned in longitudinal direction, so the absolute value of polarization is  $|P| = 1$ . Their precession around the vertical stable spin axis is tracked for 70 ms, equivalent to about 128 000 turns or 930 000 precessions. If longitudinal beam dynamics are modeled by deterministic synchrotron oscillations (blue), no depolarization occurs. Importing the stochastic synchrotron radiation model from ELEGANT (red), depolarization by spin decoherence can be simulated successfully. Tracking at other beam energies confirms, that depolarization time decreases with energy. Transverse beam dynamics have negligible impact on decoherence.

## DEPOLARIZING RESONANCES

A major challenge in operation of a synchrotron with energy ramp is crossing depolarizing resonances during acceleration without severe depolarization. Hence, the influence of the implemented synchrotron radiation on a resonance crossing is discussed here. Depolarizing resonances occur at certain beam energies, where the spin tune is in phase with some horizontal magnetic fields. Thereby, the stable spin axis is tilted into the horizontal plane.

Integer resonances appear at integer spin tune and are driven by magnet misalignments and vertical closed orbit distortions, which cause revolution harmonic field contributions. Therefore, integer resonances do not depend on betatron motion and tracking with all particles on the closed orbit does not affect the result. Figure 3 shows a POLEMATRIX example of crossing the integer resonance  $Q_{\text{spin}} = 4$  with a linear energy ramp of 4 GeV/s in the ELSA stretcher ring. The vertical polarization  $P_z$  is decreased by the resonance. Simulating without any energy spread (blue), the absolute value of polarization  $|P|$  is not affected at all. Using synchrotron radiation from ELEGANT (red), two additional effects can be observed. First, the individual synchrotron oscillations cause synchrotron side bands of the resonance. They reduce both  $P_z$  and  $|P|$ , because the crossing speed of these side bands is different for each particle resulting in inco-

herent spin motion. Additionally, after the resonance, when the polarization vector is tilted out of the vertical direction ( $P_z < |P|$ ), synchrotron radiation leads to slow depolarization, as observed before in Fig. 2.

## CONCLUSION AND OUTLOOK

The influence of synchrotron radiation on short timescale spin dynamics in circular electron accelerators was implemented successfully in the spin tracking code POLEMATRIX. This was done by importing results of the established particle tracking program ELEGANT using the lattice library LIBPALATTICE. Both LIBPALATTICE and POLEMATRIX will be published as open source software.

To improve performance, the particle tracking has to be implemented directly in POLEMATRIX. Currently, a synchrotron radiation model is tested. In the process, the ELEGANT based implementation is a reliable basis for benchmarking. Furthermore, a comparison with ZGOUBI is planned, which also includes synchrotron radiation. For that purpose, a ZGOUBI format lattice export could be added to LIBPALATTICE.

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