THERMAL AND MECHANICAL ANALYSIS OF A WAVEGUIDE TO COAX SYMMETRIC COUPLER FOR SUPERCONDUCTING CAVITIES

R. Eichhorn *, J. Robbins and V. Veshcherevich
CLASSE Cornell University, Ithaca, NY 14853, USA

Abstract
As kicks from fundamental power couplers become a concern for low emittance future accelerators, a design of a symmetric coupler for superconducting accelerating cavities has been started. In this coupler, a rectangular waveguide transforms into a coaxial line inside the beam pipe to feed the cavity. So far the RF design revealed an extremely low transversal kick but concerns about cooling and the thermal stability of the coaxial transition line remained. This contribution addresses this. We calculated the heat transfer and thermal stability of this coupler and evaluated the risk of quenching due to particle losses on the coupler.

INTRODUCTION
The low energy section of a superconducting linac can significantly contribute to the emittance increase of the accelerated beam. One component of that can be asymmetric fields coming from the fundamental power coupler. Usually, power couplers are antenna type couplers attached to a side port of the beam pipe near the cavity, usually placed on one side, generating a strong dipole kick. With a symmetric arrangement like in the Cornell injector one can avoid emittance increase due to the dipole kicks. However, quadrupole kicks remain, which guided us in designing a waveguide-to-coaxial type coupler resulting in super-symmetric fields [1]. In this paper, we will complement our earlier findings with investigations on the thermal stability of the coupler under different conditions and report on mechanical properties.

Figure 1: Cornell ERL injector cavity with symmetric coupler taken from [1]

Table 1: Parameters for Theoretical Thermal Conductivity Model Taken From [2]

<table>
<thead>
<tr>
<th>param.</th>
<th>value</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRR</td>
<td>400</td>
<td>resid. res. ratio</td>
</tr>
<tr>
<td>ρ295K</td>
<td>14.5 × 10⁻⁸ Ωm</td>
<td>res. at 295K</td>
</tr>
<tr>
<td>l</td>
<td>50 µm</td>
<td>Nb. phonon mfp</td>
</tr>
<tr>
<td>Tc</td>
<td>9.2 K</td>
<td>Nb. crit. temp.</td>
</tr>
<tr>
<td>L</td>
<td>2.45 × 10⁻⁸ W K⁻²</td>
<td>param. of Eq. 1</td>
</tr>
<tr>
<td>a</td>
<td>2.30 × 10⁻⁵ m W⁻¹ K⁻¹</td>
<td>param. of Eq. 1</td>
</tr>
<tr>
<td>B</td>
<td>7.0 × 10⁵ W m⁻² K⁻⁴</td>
<td>param. of Eq. 1</td>
</tr>
<tr>
<td>1/D</td>
<td>300 m K⁻³ W⁻¹</td>
<td>param. of Eq. 1</td>
</tr>
<tr>
<td>α</td>
<td>1.76</td>
<td>param. of Eq. 1</td>
</tr>
</tbody>
</table>

Niobium Parameters

The thermal conductivity values that we used in our simulations were taken from a combination of theoretical models and experimental data. The theoretical model that we used was the thermal conductivity equation given by [2].

\[ K_s(T) = \frac{K_{es}}{K_{en}} \left( \frac{\rho_{295K}}{L \cdot RRR \cdot T} + a T^2 \right)^{-1} + \left( \frac{1}{D e^{\frac{a}{T^2}}} + \frac{1}{B T^3} \right)^{-1} \] (1)

\[ K_{es}/K_{en}(T) \] is the ratio of superconducting to normal-conducting electron contributions to thermal conductivity. Constant parameters are given in Table 1. This model is valid for \( T < 5.8 \) K. For temperatures above 5.8 K, an experimental data set was used [3].

The specific heat of the niobium cylinder was assumed to follow the Debye model \( C_v = \gamma T + AT^3 \). Using experimental data from [4], values for the parameters were calculated as \( \gamma = 0.0946 \frac{J}{kg \cdot K^2} \) and \( A = 1.28 \times 10^{-3} \frac{J}{kg \cdot K^2} \) (for \( T > T_c \)) and \( \gamma = 0 \) and \( A = 5.01 \times 10^{-3} \frac{J}{kg \cdot K^2} \) for \( T < T_c \).

Analysis

Thermal calculations were performed using the transient thermal analysis system in ANSYS®. The front face of the cylinder was fixed at 2 K, assumed being perfectly cooled by the helium. To model the power deposition from the electron beam halo hitting the niobium surface, a heat flux of 0.5 W was applied to the rear face of the cylinder, which would be the worst case scenario.

Due to the RF field, there is additional heating along the cylinder surfaces which follows the equation

\[ \frac{dP}{dT} = \frac{1}{2} R_s |H|^2 \] (2)
where \( \frac{dP}{dA} \) is the power dissipated per unit area, \( R_s \) is the surface resistance, and \( H \) is the local magnetic field \([5]\). For \( T < T_c \), the surface resistance follows the equation

\[
R_s(T) = (2.25 \times 10^{-4}) \frac{1}{T} e^{\frac{-17.67}{T}} + 5 \times 10^{-9} \Omega \quad (3)
\]

where the first term denotes the BCS contribution at 1.3 GHz while the second the 5 \( n\Omega \) term is our assumption about the residual resistance. The magnetic field distribution was gained from RF simulations as reported in \([6]\). The magnetic field values were used to model RF heating as a temperature-dependent, position-dependent heat flux into the cylinder.

As we found that RF heating is not a factor in the thermal stability, in worst case simulations of the normal-conducting behavior of the cylinder we assumed a uniform magnetic field along the surface of 107 A/m which corresponds to the maximum field yielded by the RF simulation. Further on, we assumed a normal-conducting surface resistance of 10 \( m\Omega \).

**MECHANICAL ANALYSIS**

The mechanical modal analysis system in ANSYS® was used to find the natural frequencies at which the different components of the coupler resonate. In order to minimize the computational size of the problem, the waveguide, coaxial cylinder, front beam tube, and rear beam tube were all considered separately. The coupler was assumed to be made from niobium of RRR > 250 and its Young’s modulus was taken to be 125 GPa \([7]\).

**RESULTS**

*Thermal Analysis*

In calculations where the magnetic field values along the cylinder were not approximated, the steady state temperature distribution of the cylinder was found to not differ substantially from the case where there are no RF losses. This is because heating due to RF losses in the cylinder is on the order of \( 1 \times 10^{-3} \) W in the superconducting temperature range, so heating due to the electron beam halo dominates.

Figure 2 shows the steady state temperature distribution of the cylinder subject to RF losses and 0.5 W deposition from the electron beam halo. The maximum temperature along the cylinder at steady state is about 4.7 K (Figure 3). \( R_s \) increases substantially at \( T_c \) so heating from surface currents is much greater when the cylinder has a normal-conducting region. However, these calculations showed that regardless of the initial temperature distribution of the cylinder, the same steady state temperature distribution is reached, at least within the temperature range tested of \( T_{init} \leq 20 \) K. Simulations with greater power deposition from the electron beam halo showed that the power necessary to drive the cylinder above \( T_c \) at steady state would have to be greater than 5.5 W.

**Figure 2:** Coaxial cylinder steady state temperature distribution.

**Figure 3:** Maximum temperature along the coaxial cylinder over time for initial temperature = 20 K.

**Figure 4:** Deformation of the waveguide at the 365.54 Hz mode.
Mechanical Analysis

The five lowest frequency modes were calculated for each component. The frequencies of these modes are given in Table 2. Results for the lowest frequency mode of the waveguide and the coax cylinder are given in Fig. 4 and Fig. 5 respectively. The lowest frequency mode was calculated to be the 365 Hz mode of the waveguide. This frequency is high enough that resonance should not be dangerous.

Table 2: Lowest 5 Eigenfrequencies of Coupler Components in Hz

<table>
<thead>
<tr>
<th>wavegd.</th>
<th>coax cyl.</th>
<th>ft. beam tube</th>
<th>r. beam tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>365.54</td>
<td>868.18</td>
<td>995.09</td>
<td>2051.2</td>
</tr>
<tr>
<td>514.94</td>
<td>868.18</td>
<td>995.09</td>
<td>2052</td>
</tr>
<tr>
<td>627.9</td>
<td>966.88</td>
<td>1111.7</td>
<td>2920.3</td>
</tr>
<tr>
<td>649</td>
<td>967.07</td>
<td>1111.7</td>
<td>3582.7</td>
</tr>
<tr>
<td>730</td>
<td>1928.2</td>
<td>1914.9</td>
<td>3585.8</td>
</tr>
</tbody>
</table>

SUMMARY

The thermal calculations showed that the coaxial cylinder extending from the waveguide into the beam tube is expected to remain in a superconducting state, even when hit by particles depositing up to 5.5 W. If the cylinder fluctuates into a normal-conducting temperature range for any reason, it should return to a superconducting temperature range. The mechanical calculations showed that the lowest frequency resonant mode of the coupler should be about 365 Hz. This frequency is high enough that the coupler is not expected to be strongly mechanically excited when in use. This oscillation would not noticeably influence the RF properties. In conclusion, the coupler should not need any further cooling and is unlikely to be vulnerable to mechanical excitation.

REFERENCES