STUDY OF ORBIT CORRECTION FOR eRHIC FF AG DESIGN

C. Liu†, Y. Hao, V. Litvinenko, F. Meot, M. Minty, V. Ptitsyn, D. Trbojevic
BNL, Upton, NY 11973, USA

Abstract
The unique feature of the orbits in the eRHIC Fixed Field Alternating Gradient (FFAG) design is that multiple accelerating and decelerating bunches pass through the same magnets with different horizontal offsets. Therefore, it is critical for the eRHIC FFAG to correct multiple orbits in the same vacuum pipe for better spin transmission and alignment of colliding beams. In this report, the effects on orbits from multiple error sources will be studied. The orbit correction method will be described and results will be presented.

INTRODUCTION
Electron accelerators based on FFAG lattice are designed to be placed in the existing Relativistic Heavy Ion Collider (RHIC) tunnel for collision of electron and heavy ion beams [1]. The advantages of the FFAG lattice are in two aspects: the magnets are at fixed fields and there is strong focusing in the transverse planes. A machine with such lattice accepts beams over a large energy range. The dispersion function is very small (∼ cm) so that orbits of beams with different energies stay in the same vacuum pipe with small horizontal offsets. The orbits will be distorted differently if either the magnets are misaligned or there are magnet gradient errors. The misalignment and gradient errors in a FFAG lattice need therefore to be compensated locally to restore all the orbits to the design values. Dipole and quadrupole trims will be placed at each and every magnet center to correct the said errors. Considering the large number of magnets in the rings, a global correction scheme must detect local errors quickly and precisely. Furthermore, enough beam position monitors (BPMs) for beam position measurement is also critical to locate the errors effectively.

The lattice design has evolved much in the past year with optimization of linac size and synchrotron radiation [2, 3]. The orbit correction simulation will be presented in this paper is based on a single FFAG ring design, which accelerates electron beam from 1.9 to 10 GeV and decelerates the beam back to 1.9 GeV via energy recovery. The injected beam will be accelerated before entering the FFAG arc. Therefore, there are 9 accelerating beam passes and 8 decelerating passes through the FFAG arcs. As the beam gets accelerated, the betatron tune per FFAG cell changes from ∼0.44 to ∼0.1 in the horizontal plane and ∼0.3 to ∼0.04 in the vertical plane discretely [1].

The orbit distortion due to misalignment of magnets was studied in simulation. The orbit deviation root-mean-square (rms) in two planes for beam at 2.8GeV is shown in Fig. 1 for a range of misalignment rms. The same orbit distortion due to misalignment errors were studied for the other 8 beam passes. The magnification factors, the ratio between orbit rms and misalignment rms, are compared for all passes. The magnification factor decreases with beam rigidity as expected in the horizontal plane, however not in the vertical plane. Analytical calculation of the magnification factor (∝ √β1/β2, β1, β2 are the betatron functions at the magnets and BPMs, v is the tune per cell, γ is the Lorentz factor) confirmed the count-intuitive behavior in the vertical plane, shown in Fig. 2.

The orbit distortion due to magnet gradient errors was studied in simulation as well. The orbit rms in the horizontal plane only for beam at 2.8 GeV is shown in Fig.3 for a

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* The work was performed under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy.
† cliu1@bnl.gov

Figure 1: RMS of beam orbit distortion in eRHIC FFAG in both planes due to magnet misalignment errors, for the beam in the first pass with energy 2.8 GeV.

The measurement of beam positions is challenging for continuous bunch train with ∼ ns spacing between bunches. A gap in the electron beam bunch train, which coincides with the abort gap in RHIC, is necessary for ion clearing purpose. A diagnostic bunch will be put in the gap for routine monitoring of the beam positions continuously [4].

Figure 2: RMS of beam orbit distortion in eRHIC FFAG in both planes due to gradient errors, for the beam in the first pass with energy 2.8 GeV.

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Figure 2: Misalignment magnification factor in the vertical plane for all passes from simulation compared with theoretical calculation.

Figure 3: RMS of beam orbit distortion in eRHIC FF AG in horizontal plane due to magnet gradient errors, for the beam in the first pass with energy 2.8 GeV.

**ORBIT CORRECTION ALGORITHM**

For a linac machine with $m$ BPMs and $n$ correctors, the orbit response matrix is

$$
R = \begin{pmatrix}
R_{1,1} & R_{1,2} & R_{1,3} & \cdots & R_{1,n} \\
R_{2,1} & R_{2,2} & R_{2,3} & \cdots & R_{2,n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
R_{m,1} & R_{m,2} & R_{m,3} & \cdots & R_{m,n}
\end{pmatrix}
$$

where $R_{i,j} = \begin{cases} 
\sqrt{\beta_i \beta_j} \sin (\phi_i - \phi_j) & \text{if } \phi_i > \phi_j \\
0 & \text{if } \phi_i \leq \phi_j
\end{cases}$

The goal of orbit correction is to compensate the difference between measured and designed orbit [5],

$$
\Delta Y = (Y_0 - Y) = R \ast \theta
$$

where $Y_0$ is the target orbit, $Y$ is the measured orbit, $R$ is the response matrix, and $\theta$ is the correction strength.

Eq. 2 can be extended for the case of orbit correction using measurements from multiple passes,

$$
\begin{pmatrix}
\Delta Y_1 \\
\Delta Y_2 \\
\vdots \\
\Delta Y_m
\end{pmatrix} =
\begin{pmatrix}
R_{1,1} & R_{1,2} & \cdots & R_{1,m} \\
R_{2,1} & R_{2,2} & \cdots & R_{2,m} \\
\vdots & \vdots & \ddots & \vdots \\
R_{m,1} & R_{m,2} & \cdots & R_{m,m}
\end{pmatrix} \ast \theta
$$

$m$ is the number of passes. The measured orbit could be available for any given number (from 0 to 8) of complete accelerating passes plus any fraction of the following pass in the stage of machine commissioning. Therefore, it is the difference between available measured orbit and corresponding design orbit on the left hand side, and corresponding response matrix on the right hand side of Eq. 3.

**ORBIT CORRECTION RESULTS**

We first tested the orbit correction scheme on a single pass. One BPM was placed after each and every magnet in eRHIC lattice so that local errors could be found properly. With only misalignment errors, orbit correction was performed for the first pass only to calculate dipole corrections. Then the same set of corrections were implemented in the lattice and residual orbit distortions for the other passes were examined. The orbit distortions for all the other passes were reduced under 1 mm peak to peak. This verifies that the local errors can be located by the correction scheme properly with 2 BPMs per FF AG cell. Correction strengths calculated for different passes independently are in good agreement, which indicates local errors can be well located.

It is costly to place BPMs with all magnets in the lattice. The length of a FF AG cell is 2.58 m because of strong focusing, therefore the total number of BPMs is unpractically high. We repeated orbit correction with 1 BPM per 2 FF AG cells. The corrections calculated for a single pass in this case didn’t improve orbits for other passes. This means local errors can not be identified with the number of BPMs less than that of magnets if only the data from the first pass is used for the correction.

The tunes per cell variation for different energies is one of the reasons why orbits behave differently with errors in the lattice. On the other hand, beam position measurements for different passes all provide useful information about the sources of the errors. Therefore, one should be able to better localize the errors by correcting beam trajectories for multiple passes. The number of BPMs for an efficient orbit correction can be reduced as long as the number of measurements is greater than the number of error sources when correcting multiple orbits simultaneously.

The number of BPMs in the following simulation was set as 1 BPM per 2 FFAG cells. We assumed 100 $\mu$m rms misalignment error, 0.05 mrad rms for roll, pitch, yaw angles. The orbit correction was simulated based on six FFAG arcs in sequence with 172 cells per arc. The residual orbit errors at the end of each pass is assumed to be corrected by the
Figure 4: Horizontal orbits for 9 passes without (blue) and with (green) orbit corrections implemented for the first pass.

Figure 5: Horizontal orbits for 9 passes without (blue) and with (green) orbit corrections implemented for the first two passes.

Figure 6: Horizontal orbits for 9 passes without (blue) and with (green) orbit corrections implemented for the first four passes.

Figure 7: Horizontal orbits for 9 passes without (blue) and with (green) orbit corrections implemented for all the 9 passes.

The orbits of multiple passes in an early stage eRHIC FFAG design were studied. The orbit distortion due to misalignment and magnet gradient errors were simulated. It was concluded that the misalignment errors is the dominating source for orbit distortion. Orbit correction scheme for the eRHIC FFAG design was proposed and verified in simulation.

ACKNOWLEDGEMENTS

The authors would like to thank S. Berg, S. Brooks, I. Ben-Zvi and W. Meng for helpful discussions.

REFERENCES


SUMMARY

The orbits of multiple passes in an early stage eRHIC FFAG design were studied. The orbit distortion due to misalignment and magnet gradient errors were simulated. It was concluded that the misalignment errors is the dominating source for orbit distortion. Orbit correction scheme...