Abstract

Muon accelerator beams are commonly produced indirectly through pion decay by interaction of a charged particle beam with a target. Efficient muon capture requires the muons to be first phase-rotated by rf cavities into a train of 21 bunches with much reduced energy spread. Since luminosity is proportional to the square of the number of muons per bunch, it is crucial for a Muon Collider to use relatively few bunches with many muons per bunch. In this paper we will describe a bunch merging scheme that should achieve this goal. We present for the first time a complete end-to-end simulation of a 6D bunch merger for a Muon Collider. The 21 bunches arising from the phase-rotator, after some initial cooling, are merged in longitudinal phase space into seven bunches, which then go through seven paths with different lengths and reach the final collecting "funnel" at the same time. The final single bunch has a transverse and a longitudinal emittance that matches well with the subsequent 6D rectilinear cooling scheme.

INTRODUCTION

A high luminosity Muon Collider requires intense single muon bunches. A full scheme of a Muon Collider has been designed by the Muon Accelerator Program (MAP). In this scheme the muons are produced by a high power proton beam hitting on a target, and then they are captured and phase-rotated [1] into 21 bunches. The large initial emittance is reduced by ionization cooling, and for a high luminosity collider, the bunches need to be merged into one of each sign.

The merge concept was outlined in previous studies with preliminary simulations [2]. In this paper, we present an updated design with an end-to-end simulation using G4Beamline [3].

MERGE SCHEME

Figure 1 shows the bunch merge scheme. The incoming beam consists of 21 bunches from the output of the initial cooling channel. Each bunch has a transverse emittance ($\epsilon_T$) of 1.3 mm and a longitudinal emittance ($\epsilon_L$) of 1.7 mm. The 21 bunches are first merged longitudinally into seven bunches, by using radio frequency (rf) cavities with a series of rf harmonics. Then the seven bunches are transversely merged into one bunch. A kicker magnet kicks the seven bunches into seven "trombones" [4] (Only two trombones are shown in Fig. 1). Each trombone has a different arc length so that seven bunches arrive at the collecting section at the same time. A "funnel" [4] is designed to get the seven bunches close to each other to form a single bunch. In the end a matching section with three solenoids will match the bunch into the post-merge cooling channel.

Longitudinal Merge

The longitudinal merge uses the rf cavities with frequencies from 108 MHz to 1950 MHz to phase-rotate the bunches...
so that three bunches in a group are merged into one. This is realized with a series of rf harmonics and drift sections in five stages, as shown in Fig. 2 with phase space distributions at the beginning of each stage (a - e) and at the end of the longitudinal merge (f). The longitudinal emittance of the merged bunch is 9.7 mm with transverse emittance unchanged. The transmission in the longitudinal merge is 98% without considering muon decay and 94% with decay.

**Transverse Merge**

After the longitudinal merge, a kicker magnet separates the seven bunches to go to seven different paths. The kicker is a 3 m-long magnet with 1 m diameter. The transverse field rotates at a frequency of 18 MHz, with a strength of 220 Gauss. The first 6 bunches are kicked by 5° to six different directions and the last bunch is ideally kept in the center.

A matching section is designed using 4 solenoidal magnets to match the bunches to the "trombones". The strengths and positions of the 4 solenoids are optimized by propagating the Courant-Snyder parameters.

Table 1 gives the parameters of the matching solenoids. The kicker is at 10 m from the longitudinal merge, which means the gap between the B2 solenoid and the kicker is 0.8 m. We put the kicker close to B2 and make the distance between B2 and B3 large so that the bunches can be fanned out by the kicker with large enough space to allow seven big aperture solenoids (B3) in parallel. In this way the required kicking strength is low and the chromatic effect of the kicker is reduced.

The longest trombone which guides the first bunch is designed with four 90° bent solenoids (the first one is 85° to compensate the kicker angle). The bending radius of this trombone is 5.559 m to bring the bunch to the end of the trombone at the same time as the seventh bunch, which goes straight through the center. We use this radius for all the arcs in all trombones, because such a big bending radius requires a small bending field and causes little chromatic effect compared to small bending radius. The other trombones are designed to have smaller bending angles so that the lengths can fit the separation of the bunches, as shown in Fig. 1.

The arcs of the trombones consist of bent solenoids with dipole magnets outside. The bent solenoids have 2 T field along the centerline, to keep the bunches focused. The dipoles have 0.12 T field in the transverse direction, providing the bending force. In the middle of the trombones, there are 2 m long straight sections with rf cavities to reduce the growth of the longitudinal size of the bunches. The cavities have a frequency of 108 MHz with a gradient of 10 MV/m and are put inside the 2 T solenoids.

Figure 3 shows the evolution of $\epsilon r$, transmission, and the rms bunch length from the end of the longitudinal merge to the end of the trombone. Here only one channel with the longest trombone is shown, and the others are similar. The magnetic field along the beam line in the beam direction (Bz) is shown as the red dashed line and the transverse dipole field (Bx) is shown as the blue dotted line. The transmission (pink dashed line) without loss from decay is 93%, and the major beam losses occur in two places. The first beam loss of about 3% is caused by the relatively small aperture of magnet B3. The loss of those largely diverged muons also reduces the beam transverse emittance. The second beam loss of 3% is from the rf cavity in the trombone. The rf field cuts off the muons which have large energy divergence.

<table>
<thead>
<tr>
<th>Magnet</th>
<th>Length</th>
<th>Aperture</th>
<th>Position</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1 m</td>
<td>2.4 m</td>
<td>1.2 m</td>
<td>1.2 T</td>
</tr>
<tr>
<td>B2</td>
<td>1 m</td>
<td>2.4 m</td>
<td>7.2 m</td>
<td>0.60 T</td>
</tr>
<tr>
<td>B3</td>
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<td>24.6 m</td>
<td>0.56 T</td>
</tr>
<tr>
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<td>1 m</td>
<td>1.2 m</td>
<td>30.6 m</td>
<td>1.0 T</td>
</tr>
<tr>
<td>B5</td>
<td>1 m</td>
<td>1.0 m</td>
<td>67.5 m</td>
<td>1.2 T</td>
</tr>
<tr>
<td>B6</td>
<td>1 m</td>
<td>1.2 m</td>
<td>73.5 m</td>
<td>0.6 T</td>
</tr>
<tr>
<td>B7</td>
<td>1 m</td>
<td>1.4 m</td>
<td>83.6 m</td>
<td>0.4 T</td>
</tr>
<tr>
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<td>1 m</td>
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</tr>
<tr>
<td>B9</td>
<td>3.65 m</td>
<td>1.5 m</td>
<td>92.8 m</td>
<td>−2.6 T</td>
</tr>
</tbody>
</table>

Table 1: Magnet parameters for the matching magnets. Positions are the centers of the magnets relative to the end of the longitudinal merge.

Figure 2: Phase space distribution at locations along the longitudinal merge: a) to e) are at the beginning of each stage and f) is at the end of the longitudinal merge. The rf fields at the corresponding places are shown in blue curves. Six representative bunches are shown.
bunches first pass through 2 solenoids (B5 and B6) to reduce the angular dispersion. The parameters of these magnets are also included in Tab. 1. The funnel has 6 segments with 0.09 T dipole fields inside each of them, and conceptually the center bunch will not be affected by the field.

Figure 4 shows the transverse distribution of the bunches before and after the funnel. The seven bunches are combined to a single bunch, which has a transverse size with $\beta = 23m$. The funnel has an efficiency of 95% and most of the losses are due to the septum magnet. $\epsilon_T$ of the combined bunch is 6.5 mm, which is within the acceptance of the later cooling section.

The merged single bunch has a large transverse size and has to be matched into later cooling section. The post-merge cooling scheme will use rf cavities with wedge shaped retardation materials in an alternating magnetic field. A matching section is designed with 2 short solenoidal magnets (B7 and B8) to reduce the transverse size and another 3.65 m long solenoid (B9) to match the bunch into the alternating field. The $\epsilon_T$ conserves at around 6.5 mm and the $\beta$ function is reduced to 0.6 m, which is preferable for the cooling channel.

**TOTAL PERFORMANCE**

After the merge section, the single bunch has a $\epsilon_T$ of 6.5 mm and $\epsilon_L$ of 10.3 mm, which both reach the requirement of the later cooling channel. The transmission without decay is 87% and 78% with decay.

**SUMMARY AND OUTLOOK**

The bunch merging for a Muon Collider has been simulated from end to end. The 21 bunches from the initial cooling channel are merged into a single bunch. The bunch is matched to the later cooling channel, and the quality of the output bunch reaches the requirements for post-merge cooling.

Several technical challenges have to be investigated. The rf harmonics used in the current longitudinal merge simulation has to be replaced by rf cavities. The high frequency kicker field has to be demonstrated. The required space for cooling and cryogenic system needs to be considered in the future work.

**REFERENCES**


