THE LUMINOSITY REDUCTION WITH HOURGLASS EFFECT AND CROSSING ANGLE IN AN E-P COLLIDER

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Abstract

This paper derived the luminosity reduction caused by crossing angle and hourglass effect in an asymmetric collision. Here, we gave the general expressions of the geometrical reduction factor of luminosity for the asymmetric case caused by crossing angle and hourglass effect, for tri-Gaussian bunches colliding. We also gave it simple expression in some special cases to recover the earlier results, such as the formulas for only hour-glass effect exist and only crossing angle exist. The expressions used in e-p collider are also analysed in detail.

INTRODUCTION

An electron-ion colliders CehC based on CepC-SppC have been envisioned in China for reaching new frontiers of high energy and nuclear physics. In order to have larger collision frequency, bunch spacing is very small in the *CehC* design. This will cause the problem of parasitic collisions: bunches may interact with each other not only at the interaction point (IP) but also at points around the IP, which can be avoided by collision with a crossing angle. The proton beam have much longer bunch length than the electron beam, though both of them have the bunch length comparable to the betatron function at the interaction point, the hourglass effect is still can't neglected.

There had been several expressions of collision luminosity, which treat collision with the head-on collision of asymmetric beams with hourglass effect [1, 2], angled collision of round beams with hourglass effect [3], the angled collision of symmetric beams with hourglass effect [4] and asymmetric angled collision with hourglass effect [5]. Here, the word symmetric or asymmetric is concerned with the beam sizes of colliding beams but not the beam energies.

This paper uses the integral method to give a more general expression of the geometrical reduction factor of luminosity for asymmetric tri-Gaussian bunches angled collision with hourglass effect.

LUMINOSITY FOR ANGLED COLLISION WITH HOURGLASS EFFECT

Consider two bunches of particles with densities $\rho_1(x,y,s,t)$ and $\rho_2(x,y,s,t)$, the number of particles is N_1 and N_2, all particles are assumed to move with the common velocity $\underset{v_1}{\rightarrow}$ and $\underset{v_2}{\rightarrow}$. Then, the colliding luminosity is given by the overlap integral both in time and in space [6]

$$L = N_1 N_2 N_b f_{rev} K \int \iiint_{-\infty}^{+\infty} \rho_1 \rho_2 \,\mathrm{dxdydsdt}\,. \quad (1)$$

Where, f_{rev} is the revolution frequency, K is the kinematic factor, which is expressed as

$$\mathrm{K} = \sqrt{(\underset{v_1}{\rightarrow} - \underset{v_2}{\rightarrow})^2 - (\underset{v_1}{\rightarrow} \times \underset{v_2}{\rightarrow})^2 / c^2}\,. \quad (2)$$

x, y, s are the coordinates in the lab frame. However, the distribution functions are assumed Gaussian distributions only in the commoving frame, namely,

$$\rho_1(x_1,y_1,s_1,t) = \rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - ct)$$

$$\rho_2(x_2,y_2,s_2,t) = \rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 - ct)$$

Here,

$$\rho_{iz}(u) = \frac{1}{\sigma_{iz}\sqrt{2\pi}} exp\left(-\frac{u^2}{2{\sigma_{iz}}^2}\right) \quad i = 1,2, \quad z = x,y, \quad (3)$$

$$\rho_{is}(s - s_0) = \frac{1}{\sigma_{is}\sqrt{2\pi}} \exp\left(-\frac{(s-s_0)^2}{2{\sigma_{is}}^2}\right), \quad i = 1,2, \quad (4)$$

$$\sigma_{iz} = \sigma_{iz}^* \sqrt{1 + \frac{{s_i}^2}{{\beta_{iz}^*}^2}}\,. \quad i = 1,2, \quad z = x,y, \quad (5)$$

If there is a crossing angle Ø in horizontal (x, s)-plane in the collider, we need to transform the co-moving frame coordinates to the lab frame coordinates, shown in figure 1.

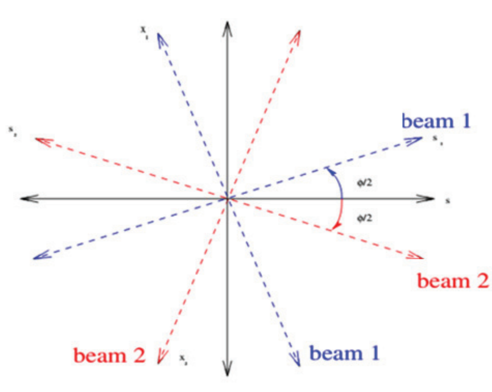

Figure1: Coordinates transformation.

$$\begin{cases} x_1 = x \cdot cos\frac{\emptyset}{2} - s \cdot sin\frac{\emptyset}{2}, s_1 = s \cdot cos\frac{\emptyset}{2} + x \cdot sin\frac{\emptyset}{2} \\ x_2 = -x \cdot cos\frac{\emptyset}{2} - s \cdot sin\frac{\emptyset}{2}, s_2 = -s \cdot cos\frac{\emptyset}{2} + x \cdot sin\frac{\emptyset}{2} \end{cases} \quad (6)$$

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Then, the luminosity is expressed

$$L = 2cos^2\frac{\emptyset}{2}N_1N_2N_bf_{rev}\ \int\iiint_{-\infty}^{+\infty} dxdydsds_0\rho_{1x}\left(x\cdot cos\frac{\emptyset}{2}-s\cdot sin\frac{\emptyset}{2}\right)\cdot\rho_{1y}(y)\cdot\rho_{1s}\left(s\cdot cos\frac{\emptyset}{2}+x\cdot sin\frac{\emptyset}{2}-s_0\right)\cdot \rho_{2x}\left(x\cdot cos\frac{\emptyset}{2}+s\cdot sin\frac{\emptyset}{2}\right)\cdot\rho_{2y}(y)\cdot\rho_{2s}\left(s\cdot cos\frac{\emptyset}{2}-x\cdot sin\frac{\emptyset}{2}+s_0\right). \quad (7)$$

Here, $s_0 = c\cdot t$, it means the distance from beam center to the colliding point.

For simplifying the integration, here, we use $s_1 = s\cdot cos\frac{\emptyset}{2}$, $s_2 = -s\cdot cos\frac{\emptyset}{2}$ since $x\cdot sin\frac{\emptyset}{2} \ll s\cdot cos\frac{\emptyset}{2}$.

After integrating over y, s_0 and x, we obtain the luminosity for the general asymmetric case caused by crossing angle and hourglass effect, for tri-Gaussian bunches colliding.

$$L = \frac{cos\frac{\emptyset}{2}\cdot N_1N_2N_bf_{rev}}{\sqrt{2}\pi^{3/2}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty}\frac{\exp[s^2\left(-\frac{1+cos\emptyset}{\sigma_{1s}^2+\sigma_{2s}^2}+\frac{cos\emptyset-1}{S_{1x}+S_{2x}}\right)]}{\sqrt{S_{1x}+S_{2x}}\sqrt{S_{1y}+S_{2y}}}ds \quad (8)$$

with

$$S_{iz} = \frac{(s^2cos^2\frac{\emptyset}{2}+\beta_{iz}^{*\,2})\sigma_{iz}^{*\,2}}{\beta_{iz}^{*\,2}} \quad (9)$$

Comparing with the general expression for luminosity

$$L = \frac{N_1N_2N_bf_{rev}}{2\pi\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}} \quad (10)$$

The luminosity reduction factor with horizontal crossing and hourglass effect for two unequal tri-Gaussian bunches is

$$Rg = \frac{\sqrt{2}cos\frac{\emptyset}{2}\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty}\frac{exp[s^2\left(-\frac{1+cos\emptyset}{\sigma_{1s}^2+\sigma_{2s}^2}+\frac{cos\emptyset-1}{S_{1x}+S_{2x}}\right)]}{\sqrt{S_{1x}+S_{2x}}\sqrt{S_{1y}+S_{2y}}}ds, \quad (11)$$

Only has Hourglass Effect

When the two Gaussian beams are head-on colliding, the horizontal crossing angle is zero, then, there only has hourglass effect, the formula (8) becomes

$$Rh = \frac{\sqrt{2}\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty}\frac{\exp\left(-\frac{2s^2}{\sigma_{1s}^2+\sigma_{2s}^2}\right)}{\sqrt{Sh_{1x}+Sh_{2x}}\sqrt{Sh_{1y}+Sh_{2y}}}ds, \quad (12)$$

$$\frac{\sqrt{Sh_{1x}+Sh_{2x}}}{\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}} = \sqrt{1+\frac{s^2\left(\frac{\sigma_{1x}^{*\,2}}{\beta_{1x}^{*\,2}}+\frac{\sigma_{2x}^{*\,2}}{\beta_{2x}^{*\,2}}\right)}{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}},$$

$$\frac{\sqrt{Sh_{1y}+Sh_{2y}}}{\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}} = \sqrt{1+\frac{s^2\left(\frac{\sigma_{1y}^{*\,2}}{\beta_{1y}^{*\,2}}+\frac{\sigma_{2y}^{*\,2}}{\beta_{2y}^{*\,2}}\right)}{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}}$$

Where, $Sh_{iz} = \frac{(s^2+\beta_{iz}^{*\,2})\sigma_{iz}^{*\,2}}{\beta_{iz}^{*\,2}}$.

Let $t = \frac{\sqrt{2}s}{\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}$, then it becomes

$$Rh = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{exp(-t^2)}{\sqrt{(1+\frac{t^2}{t_x^2})(1+\frac{t^2}{t_y^2})}}dt, \quad (13)$$

with

$$t_{x,y}^2 = \frac{2(\sigma_{1x,y}^{*\,2}+\sigma_{2x,y}^{*\,2})}{(\sigma_{1s}^{*\,2}+\sigma_{2s}^{*\,2})(\frac{\sigma_{1x,y}^{*\,2}}{\beta_{1x,y}^{*\,2}}+\frac{\sigma_{2x,y}^{*\,2}}{\beta_{2x,y}^{*\,2}})}$$

The Symmetric-Collider Case (Flat Beam)

For the symmetric-collider case, which often used in electron colliders, the word "symmetric" means

$$\sigma_{1x}^* = \sigma_{2x}^* = \sigma_x^*\ ,\quad \sigma_{1y}^* = \sigma_{2y}^* = \sigma_y^*\ , \sigma_{1s} = \sigma_{2s} = \sigma_s.$$

$$\beta_{1x}^* = \beta_{2x}^* = \beta_x^*,\ \beta_{1y}^* = \beta_{2y}^* = \beta_y^*.$$

Then, the formula (8) become

$$Rg1 = \frac{cos\frac{\emptyset}{2}}{\sqrt{\pi}\sigma_s}\int_{-\infty}^{+\infty}\frac{\exp\left[s^2\left(-\frac{1+cos\emptyset}{2\sigma_s^2}+\frac{(cos\emptyset-1)}{2\left(\frac{s^2cos^2\frac{\emptyset}{2}}{\beta_x^{*\,2}}+1\right)\sigma_x^{*\,2}}\right)\right]}{\sqrt{\frac{\left(s^2cos^2\frac{\emptyset}{2}+\beta_x^{*\,2}\right)}{\beta_x^{*\,2}}}\sqrt{\frac{\left(s^2cos^2\frac{\emptyset}{2}+\beta_y^{*\,2}\right)}{\beta_y^{*\,2}}}}ds, \quad (14)$$

When the beam is flat, with $\sigma_x^* \gg \sigma_y^*$, the hourglass effect is mainly occurred in vertical plane, let $t = \frac{scos\frac{\emptyset}{2}}{\beta_y^*}$, we can simplify the formula (14) to

$$Rg \approx \frac{\beta_y^*}{\sqrt{\pi}\sigma_s}\int_{-\infty}^{+\infty}\frac{\exp\left[\frac{t^2\cdot\beta_y^{*\,2}}{cos^2(\emptyset/2)}\left(-\frac{cos^2(\emptyset/2)}{\sigma_s^2}-\frac{sin^2(\emptyset/2)}{\sigma_x^{*\,2}}\right)\right]}{\sqrt{1+t^2}}dt = \sqrt{\frac{2}{\pi}}ae^bK_0(b) \quad (15)$$

With

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$$a = \frac{\beta_y^*}{\sqrt{2}\sigma_s}, \qquad b = a^2[1 + (\frac{\sigma_s}{\sigma_x^*}\tan\frac{\emptyset}{2})^2].$$

Where, K_0 is a Bessel function.
If $\sigma_s \ll \beta_y^*$, then there is no hourglass effect, Eq.(14) reduces to

$$Rc = \frac{cos\frac{\emptyset}{2}}{\sqrt{\pi}\sigma_s}\int_{-\infty}^{+\infty} exp\left[s^2\left(-\frac{1+cos\emptyset}{2\sigma_s^2} + \frac{(cos\emptyset-1)}{2\sigma_x^{*2}}\right)\right]ds = \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x^*}\tan\frac{\emptyset}{2})^2}} \qquad (16)$$

USING IN E-P COLLISION

Consider the case of round Gaussian bunches, $\sigma_{1x}^* = \sigma_{2x}^* = \sigma_{1y}^* = \sigma_{2y}^* = \sigma^*$, $\beta_{1x}^* = \beta_{1y}^* = \beta_1^*, \beta_{2x}^* = \beta_{2y}^* = \beta_2^*$.

If the two beams have different beam lengths, $\sigma_{1s} \neq \sigma_{2s}$, which is often occurs in e-p/A collision. The formula (11) can be expressed by

$$Rg_{ep} = \frac{2\sqrt{2}cos\frac{\emptyset}{2}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty}\frac{exp[s^2\left(-\frac{1+cos\emptyset}{\sigma_{1s}^2+\sigma_{2s}^2}+\frac{cos\emptyset-1}{\sigma^{*2}(2+\frac{s^2cos^2\frac{\emptyset}{2}}{\beta_1^{*2}}+\frac{s^2cos^2\frac{\emptyset}{2}}{\beta_2^{*2}})}\right)]}{(2+\frac{s^2cos^2\frac{\emptyset}{2}}{\beta_1^{*2}}+\frac{s^2cos^2\frac{\emptyset}{2}}{\beta_2^{*2}})}ds \quad (17)$$

Let $t = scos\frac{\emptyset}{2}\sqrt{\frac{1}{\beta_1^{*2}}+\frac{1}{\beta_2^{*2}}}$, then Eq.(17)can be changed to

$$Rg_{ep} = \frac{2\sqrt{2}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}\sqrt{\frac{1}{\beta_1^{*2}}+\frac{1}{\beta_2^{*2}}}} * \int_{-\infty}^{+\infty}\frac{\exp\left[\frac{t^2}{cos^2\frac{\emptyset}{2}\left(\frac{1}{\beta_1^{*2}}+\frac{1}{\beta_2^{*2}}\right)}\left(-\frac{2cos^2\frac{\emptyset}{2}}{\sigma_{1s}^2+\sigma_{2s}^2}-\frac{2sin^2\frac{\emptyset}{2}}{\sigma^{*2}(2+t^2)}\right)\right]}{(2+t^2)}dt \quad (18)$$

For small crossing angle, $cos\frac{\emptyset}{2} \approx 1, sin\frac{\emptyset}{2} \approx \frac{\emptyset}{2}$, the Eq.(18) becomes

$$Rg_{ep,s} = \frac{2\sqrt{2}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}\sqrt{\frac{1}{\beta_1^{*2}}+\frac{1}{\beta_2^{*2}}}} * \int_{-\infty}^{+\infty}\frac{exp[\frac{t^2}{(\frac{1}{\beta_1^{*2}}+\frac{1}{\beta_2^{*2}})}\left(-\frac{2}{\sigma_{1s}^2+\sigma_{2s}^2}-\frac{\emptyset}{2\sigma^{*2}(2+t^2)}\right)]}{(2+t^2)}dt, \quad (19)$$

If the crossing angle is totally compensated by crab cavity, there is only hourglass effect. Eq. (19) is expressed as

$$Rh_{ep}\frac{2\sqrt{2}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty}\frac{\exp\left[s^2\left(-\frac{2}{\sigma_{1s}^2+\sigma_{2s}^2}\right)\right]}{\left(2+\frac{s^2}{\beta_1^{*2}}+\frac{s^2}{\beta_2^{*2}}\right)}ds = \sqrt{\pi}\cdot u_x \cdot e^{u_x^2} \cdot erfc(u_x) \qquad (20)$$

With the expression $u_x = \frac{2\beta_1^*\beta_2^*}{\sqrt{\beta_1^{*2}+\beta_2^{*2}}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}$.

If we only consider the crossing angle, then, the Eq. (17) is simple expressed as

$$Rc_{ep} = \frac{\sqrt{2}cos\frac{\emptyset}{2}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}}\int_{-\infty}^{+\infty} exp\left[s^2\left(-\frac{2cos^2\frac{\emptyset}{2}}{\sigma_{1s}^2+\sigma_{2s}^2}-\frac{sin^2\frac{\emptyset}{2}}{\sigma^{*2}}\right)\right]ds = \frac{1}{\sqrt{1+(\frac{\sigma_{1s}^2+\sigma_{2s}^2}{2\sigma^{*2}}\tan^2\frac{\emptyset}{2})}} \qquad (21)$$

CONCLUSION

In this paper, we derived the luminosity reduction factor caused by crossing angle and hourglass effect in an asymmetric collision, when the four beta-functions and the six rms beam sizes are arbitrary, for tri-Gaussian bunches colliding. We also present the expressions of specific application in some special case, like the case of two Gaussian beams are head-on colliding, the symmetric-collider case with $\sigma_x^* \gg \sigma_y^*$ and the e-p collider case.

The result can be used in almost colliders without consider the offset in interact point. Especially the e-p collider case, which there is not a luminosity reduction expression for us.

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