STUDY OF BEAM-BEAM EFFECTS IN FCC-he

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Abstract
Beam-beam effects of the ring-ring scheme of FCC-he and LHeC are being studied using weak-strong simulations. The beam-beam tune shift of the electron beam is one order larger than that of proton beam. The study of the electron motion under the beam-beam interaction is the main subject. Luminosity and equilibrium beam size and beam lifetime are analysed.

INTRODUCTION
Proton (hadron)-electron collision is one of the operation modes of FCC. Either an ERL or a storage ring is considered for the electron beam accelerator. In this paper we focus on the storage ring, i.e. the so-called ring-ring scheme. The electron beam collides with proton beam with energy \( E = 50 \) TeV. The shape of the FCC proton beam is close to round, with equal emittances in both transverse planes. The electron beam should have the same beam size at the collision point. The emittance of the proton beam is very small, \( \varepsilon = 0.04 \) nm, \( \beta_{sxy} = 0.4/0.1 \) m for the proton beam gives the IP beam size \( \sigma_{sxy} = 4/2 \) \( \mu \)m. On the other hand, the rms bunch length of the proton beam is very long, i.e. 8 cm, to be compared with only 1-2 mm for the electron beam. To optimally match the beam sizes at IP, the choice of the electron-beam emittance and \( \beta_{sxy} \) is multi-faceted. Strong hourglass effect appears for \( \beta_{sxy} \) squeezed to values smaller than the proton bunch length. The allowed synchrotron-radiation power of 50 MW limits the total bunch intensity of electron beam. The beam-beam tune shift of proton beam is rather small, while that of electron beam tends to be large. We can choose either \( \beta_{sxy} \sim \sigma_{x} \) or \( \beta_{sxy} \ll \sigma_{x} \). The study of the beam-beam interaction for large beam-beam tune shifts in a weak-strong model is the main subject of this paper.

BEAM-BEAM SIMULATION METHOD
We are using weak-strong simulations in which the proton beam is represented by a fixed Gaussian distribution of macro-particles, that is, the proton and electron beams are regarded as the strong and weak beams, respectively.

The proton beam (bunch) is sliced into 100-200 pieces longitudinally. The number of pieces required depends on the ratio of \( \sigma_{x}/\beta_{x} \). The electro-magnetic field of a proton beam traveling at the speed of light is formed in the plane perpendicular to the traveling direction. The electro-magnetic field of each slice depends on the charge in a slice of thickness \( dz \) and on the distribution (Gaussian in x-y plane). The motion of the weak beam particles is modelled by applying kicks corresponding to the integrated effect of the electro-magnetic field per slice followed by drifts between slices. The kick, which a charged particle with a deviation of \((x,y)\) from the center of the distribution experiences, is expressed using Bassetti-Erskine formula [2]. The beam size \( \sigma_{x}(s) \) where electron particle \((z)\) collides with a proton slice \((z)\) depends on the collision point \( s = (z-\overline{z})/2 \). \( \sigma_{x}(s) \) is determined by the beta function variation near the IP. A longitudinal kick is applied to guarantee the symplecticity [3]. The beamstrahlung is also taken into account [4, 5].

LUMINOSITY SIMULATION FOR FCC-he AND LHeC
Simulations are performed using 10,000 macro-particles for the luminosity calculation [5]. The collision range of two beams with bunch length \( \sigma_{zp} \) (protons) and \( \sigma_{ze} \) (electrons) is \( s \approx \pm (\sigma_{zp} + \sigma_{ze}) \approx \pm \sigma_{zp} \). The ratio between proton bunch length and electron IP beta function \( \beta_{ze} \) is \( \sigma_{zp}/\beta_{ze} \approx 10 \) at 120 GeV or 20 at 60 GeV. The area \( s=\beta_{ze} \) is divided into 10 steps to ensure a good convergence of the simulation. The total number of bunch slices \((z)\) is chosen 100 (120 GeV) and 200 (60 GeV). The simulations are performed over 2,000 and 20,000 turns for 120 and 60 GeV, respectively. These simulation periods correspond to 2000/144=14 times, or 20,000/1,152=17 times, the radiation damping time, respectively. The transverse tune is chosen as \((\nu_{x}, \nu_{y})=(0.54,0.61)\), which has been found to be the best working point for FCC-ee [6]. The synchrotron tune is chosen as 0.025.

Luminosity and beam sizes of the electron beam are evaluated turn by turn. Figure 1 shows the evolution of luminosity. The luminosity drops very quickly in collisions for both 120 GeV and 60 GeV \( \varepsilon_{c} \), much below the design values of \( 7 \times 10^{33} \) and \( 6.2 \times 10^{33} \) cm\(^{-2}\)s\(^{-1}\), respectively.
Figure 1: Evolution of luminosity. Left and right plots depict the simulated luminosity for collisions with 120 GeV and 60 GeV $e^-$, respectively.

The evolution of beam sizes is shown in Figure 2. The transverse sizes and bunch length are plotted in the left and right pictures, respectively. The transverse beam sizes increase very quickly from the design values, which are 4 $\mu$m (x) and 2 $\mu$m (y). On the other hand, the bunch lengths stay at the design values, 1.2 mm (120 GeV) and 1.5 mm (60 GeV), which shows that the effect of beamstrahlung is not strong in FCC-he.

Figure 2: Simulated beam size evolution. The transverse sizes and the bunch length are plotted in the left and right pictures, respectively.

The strong luminosity degradation is caused by the hourglass effect, related to the large ratios $\sigma_{zp}/\beta_{ye}$~10 (120 GeV) and 20 (60 GeV). The high beta area of the electron beam dominates in the electron tune shift. The latter gets as high as $(\Delta \nu_x, \Delta \nu_y)=(0.85,2.893)$ and $(3.175,11.86)$ for 120 GeV and 60 GeV, respectively.

One possibility to relax the high tune shift, is adopting collisions with a finite crossing angle ($\theta_c$). For $\theta_c \beta_{ye}/2\sigma_x$~1, the collision area is limited $z \approx \pm \beta_{ye}$, and, thereby, the tune-shift contributions from the high-beta area are avoided. Figure 3 shows the geometrical and equilibrium luminosities as a function of crossing angle. The three nonzero angles in the plots correspond to $\theta_c \beta_{ye}/2\sigma_x$ =0.5, 1.0, 2.0. Increasing the crossing angle, the geometrical luminosity decreases, but the equilibrium luminosity does not change remarkably. There is no gain for increased crossing angle, though the tune shift is relaxed.

Figure 3: Initial and final luminosities given by the simulation in Fig.4 as function of crossing angle.

We next study the effect of higher $\beta^*$ while keeping the same beam sizes, i.e. the emittances are reduced as 1/$\beta^*$. Though the hourglass effect is relaxed, the tune shift increases in proportion to $\beta^*$. We look at the following cases:

- $E=120$ GeV $\epsilon_{xy}$ =0.094/0.047 nm, $\beta^*_{xy}$ =0.17/0.085 m, $\Delta \nu_{xy}$ =1.41/1.59
- $E=60$ GeV $\epsilon_{xy}$ =0.19/0.095 nm $\beta^*_{xy}$ =0.08/0.04 m, $\Delta \nu_{xy}$ =1.56/2.41.

These values correspond to 10 times higher $\beta^*$ and 1/10 times smaller $\epsilon$. The tune shifts are huge, but they are smaller than those of the design. Figure 4 shows the simulated evolution of luminosity and transverse beam size. Again no change in the bunch length is seen, while the luminosity increases drastically. The transverse emittance increase is comparable to the one obtained for nominal beta/emittance. However the geometrical luminosity is higher thanks to the smaller hourglass loss.

Figure 4: Evolution of luminosity and transverse beam size for ten times higher beta along with 10× smaller $\epsilon$.

Beam tail and lifetime should be concerned in such collision with the high beam-beam parameter. Figures 5 and 6 shows the beam tail distribution in transverse amplitude. The tail distributes by 20 $\sigma$. Physical aperture should be designed to accept this tail.
The same simulations are executed for LHeC [7]. Figure 7 shows the corresponding evolution of luminosity and transverse beam size. Once more no change in the bunch length is seen. Vertical beam size somewhat increases for both cases of High Acceptance Layout (HA) and High Luminosity Layout (HL) [7, 8]. The exact behaviour depends on the operating point in the tune plane. Final luminosity is close to the design value of $7.3 \times 10^{32}$ (HA) and $1.3 \times 10^{33} \text{ cm}^{-2} \text{s}^{-1}$ (HL).

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**REFERENCES**


[5] BBWS code developed by the author (K. Ohmi) is used for the simulations. The code treats 6 dimensional motion of colliding beam particles.

