BEAM DYNAMICS STUDIES TO DEVELOP A HIGH-ENERGY LUMINOSITY MODEL FOR THE LHC

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Abstract

Luminosity, the key figure of merit of a collider as the LHC, depends on the brightness of the colliding beams. This makes the intensity dependent beam-beam effect the dominant performance limiting factor at collision. The parasitic interactions due to the electromagnetic mutual influence of the beams in the interaction region of a collider induce a diffusive behaviour in the tails of the beam. The evolution of charge density distribution is studied to model the beam tails evolution in order to characterize beam lifetime and luminosity. To achieve this, tools are developed for tracking distributions of arbitrary number of single particles interacting with the opposing strong-beam, to analyse the halo formation processes due to the combined effect of beam-beam and machine non-linearities. This paper presents preliminary results of the simulations, both for the LHC Run I and nominal LHC parameters. The former will be used to benchmark simulations while the latter aims at supporting luminosity estimate for the Run II.

INTRODUCTION

The figure of merit of a collider is the (instantaneous) luminosity \( \mathcal{L} \) defined as the proportionality factor between the cross-section \( \sigma_p \) and the rate of events per seconds \( \frac{dN}{dt} \), and corresponds to the volume of overlap integral of the beams charge density distributions in the interaction region times the number of bunches, the revolution frequency and the product of the population in each of the opposing bunches. In the simplifying assumption of round (\( \sigma_x = \sigma_y \)), symmetric, equally populated beams colliding head-on and with Gaussian charge density distribution, the ideal luminosity formula is the following

\[
\mathcal{L}(t) = \frac{n_b f_{rev} N^2(t)}{4 \pi \sigma_x(t) \sigma_y(t)} = \mathcal{L}_0(t)
\]

where \( N \) is the number of particles in each of the colliding bunches, \( \sigma_x, \sigma_y \) are the beam transverse size, \( \gamma_{rel} = (1 - (\frac{\beta}{\gamma})^2)^{-1/2} \) is the relativistic gamma, \( \varepsilon \) is the normalised transverse emittance, \( n_b \) is the number of bunches per beam and \( f_{rev} \) is the revolution frequency.

In the real machine additional effects reducing luminosity are present: the presence of a crossing angle, the hour-glass effect due to the spatial modulation of the betatron function when \( \sigma_x \gg \beta^* \), where \( \sigma_x \) is the bunch length, and an offset in the position of the beams at the interaction point. Thus, a more realistic expression for the instantaneous luminosity is [1]

\[
\mathcal{L}(t) = \Lambda(t) N^2(t) = \frac{n_b f_{rev} \gamma_{rel} \varepsilon}{\beta \varepsilon_m} H(\sigma_x(t), \beta^*) F(\beta, \sigma_x(t), \beta^*)
\]

where \( H(\sigma_x(t), \beta^*) \) is the reduction factor due to the hour-glass effect and \( F(\sigma_x(t), \beta^*) \) is the geometrical reduction factor due to the presence of a crossing angle \( \theta_c \) that reduces the geometrical intersecting volume of the colliding beams with respect to the head-on one. This form of the expression of luminosity is convenient to highlight its dependence on beam intensity, while the other factors are included in the \( \Lambda(t) \) term.

It is evident how, for a fixed energy value, such as the collision one, the easiest way to increase the luminosity would be to increase the ratio of \( \frac{N}{\varepsilon} \), which is proportional to the beam brightness. From injection to collision energy, several phenomena interplay to cause emittance growth, and in particular intra-beam scattering (IBS). Once the beams reach the maximum energy level and are brought into collision, the intensity dependent long-range beam-beam effect is dominant, putting a limit to performance optimization. This phenomenon is due to the non-linear lens behaviour of the electromagnetic field of one beam on the other one in the interaction region, both in the collision point (head-on) and at a distance from the design interaction point (long-range).

The parasitic encounters drive beam-halo formation. Our aim is to study the beam dynamics of the halo through numerical simulations, to characterize beam lifetime and luminosity evolution at high energy. To reach this goal, the evolution of charge density distributions in the beam is analyzed, by tracking particles through a symplectic integrator that includes the beam-beam effect as a thin-kick in the transverse phase-space. The final goal is to find ways to better characterise the beam lifetime and be able to predict the component of luminosity evolution due to beam-beam effect through numerical simulations based on detailed beam dynamics models.

SIMULATIONS PARAMETERS

We have used MAD [2] to generate the LHC machine optics from Run I and the nominal LHC parameters, with

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run I</th>
<th>nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{kin} ) [TeV]</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( \varepsilon_n ) [\mu m-rad]</td>
<td>2.5</td>
<td>3.75</td>
</tr>
<tr>
<td>( \beta_s ) [cm]</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>bunch spacing [ns]</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>( \Delta \rho )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( N )</td>
<td>1.6</td>
<td>1.15</td>
</tr>
</tbody>
</table>
beams at collision energy and crossing angle active in all the four Interaction Points (IPs). Table 1 summarises the parameters used in the simulations we will present in this paper.

The number of parasitic encounters in the interaction region, and their strengths, are generated through a dedicated subroutine for the different bunch-by-bunch (bbb) separation and an additional fixed conservative number $n_{D1} = 5$ of parasitic encounters in the separation dipoles are added to it. The optics model includes the presence of multipole normal and skew errors of order $3 \leq n \leq 11$. The seed number to generate their values is chosen randomly, and is kept constant across all simulations. As the nominal LHC bunch spacing is smaller than in the Run I parameters, we expect to have more beam-beam encounters. On the other hand, beam charge is higher in Run I parameters than in the nominal ones, leading to stronger beam-beam kicks. Simulations aim also at understanding which of the two phenomena influences more the behavior of the beam.

The distribution of particles has then been tracked along the machine with SixTrack [3] that allows studying six-dimensional motion of individual particles (weak-beam) interacting with the potential generated by the other (strong) beam. We have tracked their behaviour across the machine for $10^5$ turns, corresponding to approximately 9 seconds within the LHC. The initial conditions have been assigned so that every group of 60 particles represents a portion of a bigger uniform particle distribution. Tracking a uniform distribution instead of a Gaussian enables to sample more effectively the particles in the halo, which would be highly under-sampled otherwise. A charge density weight is then assigned to each of the particles, which is the value of a transverse bivariate-Gaussian function corresponding to the particle initial position in configuration space. We follow the determination of the core-halo limit as proposed in [4], in the hypothesis of a stable round Gaussian beam. Moreover, we are not interested in studying the particles that have betatron amplitudes bigger than the tertiary collimators aperture, as at this stage collimators would have scraped them all. Taking into account this, the initial conditions for the tracking of the particles in the distribution are within the following boundaries:

$$\sqrt{3}\sigma \leq r = \sqrt{x_0^2 + y_0^2} \leq 8.3\sigma, 9\sigma$$

The maximum value are chosen as they are the collimators half-gaps widths at IP1 and IP5 for the nominal LHC and Run I, respectively [5]. The r.m.s. beam size is defined for a normalized emittance of $3.5 \mu m$-rad.

The expression of the transverse beam-beam kick on the test particle when traveling through the opposing beam electromagnetic field (weak-strong regime) is calculated in SixTrack following Basetti-Erskine [6].

**SIMULATIONS DATA ANALYSIS**

Figure 1 shows the initial conditions of the particles forming the uniform distribution that we have been tracking in the two machine/beam configurations. The 6240 samples are uniformly distributed in configuration space. The closed-orbit is subtracted from the coordinates in 4D phase-space. The initial transverse angle is zero for all the particles, as this is a parameter that does not influence the dynamics.

![Figure 1](image1.png)

**Figure 1**: Initial conditions for nominal LHC parameters of the particles tracked with SixTrack. For each plot, the subplots show transverse configuration space (top left), action space (top right), horizontal phase space (bottom left) and vertical phase space (bottom right). The colorbar indicates the value of the bivariate Gaussian distribution as a function of the amplitude of oscillation of each macroparticle.

![Figure 2](image2.png)

**Figure 2**: Top: run I parameters, bottom: nominal LHC parameters. Percentage of lost beam charge calculated summing up the different weights associated to the lost particles. Run I parameters induce a bigger charge loss as the beam-beam kicks are stronger because of the higher intensity.
Figure 3: Comparison between percentage of lost charge (top subplot) and percentage of lost particles (bottom subplot) as a function of time for Run I (up) and nominal (down) LHC parameters.

Figure 4: Comparison between percentage of lost particles with a certain initial condition for Run I (top) and nominal (bottom) LHC parameters.

Figure 2 shows the relative beam charge losses, in percentage of total beam charge, for the two parametric cases. The intensity of the beams in Run I configuration is higher than in the nominal LHC and the emittance is also smaller, so the beam-beam kicks are stronger and we would expect to see more particle losses in Run I case. The simulations indeed show that the final value of losses is higher for Run I parameter. The evolution in time, nevertheless, is different in the two cases. This is why numerical simulations are needed, although it is computationally expensive to track the particles for more than $10^5$ turns.

Finally, Figure 3 shows the turn by turn relative charge density and particle losses, as a checkpoint that the weights calculation is consistent. Where there are peaks in the intensity losses but not in the particle losses, it means that a particle closer to the core is lost, and vice versa.

The fact that losses are so small can be explained by a histogram of number of lost particles versus initial position (including closed orbit). In fact, as shown in Figure 4, there are no losses before $7\sigma$ in either cases.

SUMMARY AND FURTHER DEVELOPMENTS

A way to track particles distributions using SixTrack tracking code has been described and the first results of numerical simulations based on detailed beam dynamics have been presented to estimate the beam-charge losses in the halo made of single particles interacting at collision in a weak-strong regime of beam-beam interaction. This study is the first step to further understand the phenomena that govern luminosity and beam lifetime at collision for the LHC. Further developments foster to better characterize the impact of the error seed number by analyzing dynamic aperture studies and to extend the time interval of the simulations to enable comparison with data from past and future LHC runs.

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REFERENCES