In this work we present a preliminary study of a long chain of cavities and some aspects involved in the optimization procedure. It is important to numerically model and optimize the SRF cavities with respect to external quality factors of the most dangerous higher order modes. BESSY-VSR is an upgrade scheme for the existing BESSY II storage ring aiming to simultaneously support variable electron pulse lengths. Currently, BESSY II supports long 15 ps bunches in the standard user optics configuration and short 1.5 ps bunches in a so-called low-\(\alpha\) optics mode. In order to develop BESSY II into a variable electron pulse length storage ring, additional two sets of SRF higher-harmonic cavities will be installed. The present RF acceleration system operates at 0.5 GHz and the additional 3rd harmonic and 3rd sub-harmonic cavities will operate at 1.5 GHz and 1.75 GHz, respectively. These cavities are essential to produce short 1.5 ps bunches with a design current of up to 0.8 mA per bunch. The total current in the storage ring is limited by the higher order mode damping capabilities of the SRF cavities.

END-GROUP OPTIMIZATION

The design of the cavity used in this work is similar to the bERLinPro \cite{1} energy recovery linac main cavity. It is characterized by spline-nose cavity to beam-pipe transition and threefold waveguide higher order mode (HOM) couplers. One of the waveguide HOM couplers is replaced by a coaxial input coupler as shown in Fig. 1. The Cornell mid-cell based on TESLA design and used in the bERLinPro cavity was selected for this study. The mid-cell design was scaled to the operational frequency of 1.5 GHz and 1.75 GHz. The results of a comparison between Cornell mid-cell and other mid-cell designs were discussed previously elsewhere \cite{2}.

Field Flatness in Cavities

Usually, in the optimization procedure of the end-groups of a superconducting radio frequency (SRF) cavity one can aim at multiple optimization targets like minimum magnetic and electric surface fields, operational frequency, maximum accelerating gradient \((R/Q)\) and field flatness. In this work the optimization focused only on the operational frequency and field flatness. Typically the \(E\)-field flatness is of a concern. The \(E\)-field flatness on the beam axis is defined as \cite{3}

\[
\eta_{E_z} = \left( 1 - \frac{\sigma_{E_z,\text{peak}}}{\mu_{E_z,\text{peak}}} \right) \times 100\% \tag{1}
\]

where \(\mu_{E_z,\text{peak}}\) is the mean value of the peak electric field component \(E_z(z)\) on the beam axis in the direction of the beam in every cell and \(\sigma_{E_z,\text{peak}}\) is the standard deviation.

The BESSY-VSR higher harmonic cavities will not operate close to the maximum achievable electric field gradient, thus the electric peak surface fields and field assisted emission become less of a concern. This allows to focus on peak magnetic surface fields that contribute to magnetic quenches. The \(H\)-field flatness is even more of an issue in cavities with mid-cell designs that have relatively large iris

Figure 1: Example of 5-cell 1.5 GHz cavity model. The blue profile lines along the beam axis and along the cavity’s boundary are used to evaluate \(E\)- and \(H\)-field patterns, respectively.

Figure 2: Normalized \(E\)- (solid blue line) and \(H\)-field (dashed red line) patterns along profiles depicted in Fig. 1 for 1.5 GHz cavity.

Figure 3: Normalized \(E\)- (solid blue line) and \(H\)-field (dashed red line) patterns along profiles depicted in Fig. 1 for 1.75 GHz cavity.
radii \[4\]. One can define \(H\)-field flatness

\[
\eta_{WH} = \left(1 - \frac{\sigma_{WH,i}}{\mu_{WH,i}}\right) \times 100\%,
\]

where \(\mu_{WH,i}\) is the mean value of the magnetic energy contained in every cell, with \(W_{H,i} = \int_{V_i} w_{H,i} dV\) the magnetic energy and \(w_{H,i}\) the magnetic energy density in the \(i\)-th cell and \(\sigma_{WH,i}\) the standard deviation.

Particle swarm algorithm in CST Microwave Studio (MWS) \[5\] was used to optimize the cavities. The number of particles in the swarm was set to 15 and the maximum number of iterations was 20, resulting in 300 solver evaluations per optimization run. The goal function of the optimizer was defined as follows: The frequency of the \(\pi\)-mode should be as close as possible to 1.5 GHz or 1.75 GHz with weight 1; magnetic energy contained in every cell should be as close as possible to 0.2 J with weight 1. Field amplitudes of the eigenmode solutions in CST MWS are normalized to 1 J total stored energy. Therefore, in a 5-cell cavity the amount of stored magnetic energy is around 0.2 J per cell. The optimization was performed over two parameters, the end-cell and the mid-cell equator radii \(r_{\text{eq},m}\) and \(r_{\text{eq},e}\), respectively. In both cases, 1.5 GHz and 1.75 GHz, the end groups are symmetrical. The parameter range of \(r_{\text{eq},m}\) and \(r_{\text{eq},e}\), over which optimization was performed, was set to \(\pm 0.1\%\) of pre-tuned \(r_{\text{eq},m}\) and \(r_{\text{eq},e}\) anchor values. In this case the pre-tuning process is a simple parameter sweep over \(r_{\text{eq},e}\) resulting in the correct frequency of the fundamental \(\pi\)-mode.

In the models the beam-pipe radius is \(r_{\text{BP}} = 55\) mm, like in bERLinPro \[1\] case. The HOM waveguide couplers have width \(a_{\text{WG}} = 80\) mm and height \(b_{\text{WG}} = a_{\text{WG}}/2 = 40\) mm. Due to considerations on the cutoff frequency of the first waveguide mode \(f_{\text{co},1}\), the maximum size that can be used is limited by the 1.75 GHz cavity. The waveguide’s \(f_{\text{co},1}\) must be well above 1.75 GHZ and below the first dipole band. The size of HOMs waveguide couplers close to the 1.5 GHz cavity can be extended up to \(a_{\text{WG}} = 90\) mm and \(b_{\text{WG}} = 47.5\) mm. However extended studies on a chain of cavities must be performed to ensure lack of coupling between 1.75 GHz cavity accelerating \(\pi\)-modes and the HOM waveguide couplers.

Figure 2 and 3 present results of the 1.5 GHz and 1.75 GHz cavity optimization, respectively. In both cases, the \(E\)-field profile is on the beam axis (\(z\)-axis) and the \(H\)-field profile is along the boundary of the cavity and the spline-nose transition to the beam-pipe, as depicted in Fig. 1. It can be seen that the \(H\)-field profile slightly differs on the left and the right sides of the cavities. This effect is more prominent for the 1.75 GHz cavity. To some extent, it could be compensated with asymmetric end-group optimization, where end-cells on both sides of the cavity have different equator radii. However, this contributes to much longer optimization times due to bigger number of solver evaluations necessary to find a good solution. The \(H\)-field flatness values obtained during the optimization of cavities are listed in Table 1. The field flatness results could also be improved with asymmetric end-group optimization. The selected beam-pipe radius \(r_{\text{BP}} = 55\) mm contributes significantly to the \(E\)-field leakage. Again, this effect is more prominent for the 1.75 GHz cavity. The cavities were used to create a model of a chain of four cavities.

**Chain of Cavities**

The total length of the presented model of the chain of four cavities is 3.814 m. It is very close to 4.2 m, the
Table 1: Single Cavities

<table>
<thead>
<tr>
<th>Cavity</th>
<th>( \eta_Ez [%] )</th>
<th>( \eta_WH [%] )</th>
<th>( R/Q [\Omega] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 GHz</td>
<td>98.51</td>
<td>99.56</td>
<td>545.6</td>
</tr>
<tr>
<td>1.75 GHz</td>
<td>98.57</td>
<td>99.28</td>
<td>540.5</td>
</tr>
</tbody>
</table>

Table 2: Cavities in a Chain

<table>
<thead>
<tr>
<th>Cavity</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f [GHz] )</td>
<td>1.75</td>
<td>1.75</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>( \eta_Ez [%] )</td>
<td>97.59</td>
<td>97.64</td>
<td>98.13</td>
<td>98.14</td>
</tr>
<tr>
<td>( R/Q [\Omega] )</td>
<td>535.1</td>
<td>560.4</td>
<td>552.6</td>
<td>551.5</td>
</tr>
</tbody>
</table>

length available in a straight where the cryomodule will be installed at BESSY II storage ring. Thus, the model can be representative of issues that can be encountered when cavities are put to close together. The model shown in Fig. 4 consists of (from left to right) two 1.75 GHz cavities and two 1.5 GHz cavities. Second and fourth cavities are rotated by 180° with respect to its neighbours in order to reduce coupler kicks. Even though the same optimized single cavities were used in the model of the chain, small differences in operational frequencies of the fundamental modes are observed. The difference in surrounding volumes of each cavity contribute to splitting of eigenfrequencies of the accelerating modes. The modes are not recognized as a single one by the eigenmode solver. This purely numerical effect depends on the beam-pipe length between cavities, i.e. if the beam-pipes were shorter one would obtain only a single eigenmode for each type of cavities. The difference in frequencies of the fundamental \( \pi \)-modes is in range of 0.2 kHz for 1.5 GHz cavities and 0.01 kHz for 1.75 GHz cavities. The effect is much smaller for 1.75 GHz cavities due to bigger evanescent length of the \( \pi \)-modes in the beam-pipe. If the cavities will be arranged in a superstructure [6], this effect will disappear. It can be observed in Fig 5 that the \( E \)-field patterns in the chain are also slightly different than for single cavities. The contribution to the \( E \)-field pattern in the first (from left) 1.75 GHz cavity from the mode belonging to the second cavity 1.75 GHz cavity is not negligible. Similar situation occurs for 1.5 GHz cavities, although the effect is two orders of magnitude lower due to a smaller evanescent coupling length. The effect on \( E \)-field flatness is noticeable as shown in Table 2. This effect will be magnified in a superstructure [6]. Hence, a chain-wise optimization of end-groups of cavities is necessary.

**HIGHER ORDER MODES**

Damping of HOMs in SRF cavities is an important issue in modern high beam current storage ring applications like BESSY-VSR [7]. The HOMs can lead to coupled bunch instabilities (CBIs) [8]. Thus, it is essential to understand the geometrical influences on properties of HOMs in single cavities as well as in chains. Previously presented in [2], the dependence of external quality factors (\( Q_{ext} \)) of HOMs on the mid-cell design is only one aspect that needs to be taken into account. Various geometrical parameters can influence the \( Q_{ext} \) of HOMs, for example the size of waveguide couplers [4].

Parameter sweeps over \( r_{BP} \) have been performed for 1.5 GHz and 1.75 GHz cavities to check the influence of the HOM waveguide couplers on \( Q_{ext} \). For 1.5 GHz cavity \( a_{WG} = 80 \) mm and \( a_{WG} = 90 \) mm was used, and for 1.75 GHz cavity \( a_{WG} = 70 \) mm and \( a_{WG} = 80 \) mm was used. Fast reduced order model frequency domain solver was employed in MWS. The \( Q_{ext} \) factors of all modes in frequency range from 1.4 GHz to 3.9 GHz were extracted from transmission S-parameter spectra by means of vector fitting [9]. Figure 6 shows second transverse magnetic (TM) monopole band, from which modes highly contribute to CBIs, for a 1.75 GHz cavity with \( a_{WG} = 80 \) mm. It can be seen that the \( Q_{ext} \) experience minimum values for \( r_{BP} = 50 \) mm. A similar effect occurs for the first transverse electric (TE) quadrupole band, which usually has much higher \( Q_{ext} \). The effect does not occur for the first and the second dipole bands. These bands also highly contribute to CBIs [8] but their \( Q_{ext} \) is independent of changes in \( r_{BP} \) range from 46 mm to 55 mm.

**CONCLUSIONS**

The preliminary study on a long chain of cavities has been conducted. Asymmetric end-group optimization of SRF cavities is advantageous although numerically expensive. If used in a chain-wise optimization, it may lead to very long computation times. Estimation of \( Q_{ext} \) factors of HOMs in chain of cavities is necessary to provide full overview of HOM damping capabilities. These aspects should be taken into account during a final design optimization stage.

**ACKNOWLEDGMENTS**

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