

EMITTANCES OF THE CORE AND OF THE HALO

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Abstract

In high intensity accelerators, the beam is often space charge dominated. The density profile then takes a shape very different from a Gaussian one, with a more or less sharp core and a more or less compact halo. Furthermore, the core and the halo can be differently focused and thus differently oriented in the phase spaces. In these conditions, classically characterizing the beam by a global set of RMS values, namely Emittance and Twiss parameters, is not enough meaningful. This paper extends the core-halo limit defined earlier in 1D real space [1] to the 2D phase space, allowing to define for the very first time Emittances and Twiss parameters for the core and the halo separately. Applications to some typical beam distributions are given as an example of more appropriate beam characterization for high intensity linacs.

THE CORE-HALO ISSUES

In high intensity linacs, the space charge induces strong self-repelling electric forces on the particle beam. These nonlinear internal forces can lead to emittance growth and halo formation, two linked phenomena leading to an increased beam size and particle losses on the beam pipe. As beam power increases with beam energy and intensity, the beam power rapidly increases in high intensity linacs; furthermore, with increasing halo proportion, the power stored in it can become significant. A special attention should thus be brought to the halo behaviour and not only to the beam core. Halo quality is also important in high intensity machines.

A common practice in the design or the tuning of an accelerator is to minimize global emittance growth, but halo quality is also important in high intensity machines. Halo scraping equipment is frequently installed to limit the extension of the halo. However a widespread definition of what and where the halo is has yet to be agreed upon [2]. It is also true that a definition for halo has to suit each experiment's application, constraint and typical beam profile.

Regarding beam dynamics, the central dense core and the surrounding thinner halo are subject to different space charge force regimes [3]. As a consequence their dynamics are different and need to be studied separately.

Gaussian-like beam distributions are very common in accelerators and defining the halo as particles further than a certain number of standard deviations in this case is only natural. In high intensity linacs though, the low energy and thus high perveance makes the beam sensible to space charge effects. As a consequence beam distributions are often far from Gaussian.

A precise determination of the core-halo limit is thus needed and was proposed in [1, 4]. Such a limit allows to

know whether a particle is part of the halo or not and to study the specific characteristics of the core and the halo.

A PRECISE DETERMINATION OF THE CORE-HALO LIMIT

We will here recall the one dimensional criterion defined to determine the precise location of the core-halo limit. This criterion originated from considering the extreme case of a uniform core surrounded by a tenuous halo, where the core-halo limit is indisputably at the foot of the discontinuity in density ρ , i.e. where there is an infinite change in the slope of ρ . For a more general case of continuously varying density, it is proposed that the core-halo limit is where there is the biggest change in slope, that is where the second derivative ρ'' is maximal [1, 4]. Once this limit precisely known, it is then possible to define the parameters PHS and PHP, which are respectively the Percentage of Halo Size and Percentage of Halo Particles.

The interest of such a limit is that it corresponds to a transition from two different space charge fields [3].

A TWO DIMENSIONAL EXTENSION OF THE METHOD

The work presented here is an extension of the previously mentioned criterion to two dimensional distributions $\rho(x,y)$ in any phase space. The general algorithm used to determine such a two dimensional core-halo limit is detailed and applications to some examples are displayed.

The two dimensional distribution is first converted into a two dimensional histogram. The "good" number of bins is given by: $NbBins = SizeRatio * \sqrt[4]{NbParticle}$, where *SizeRatio* was fixed to 3.0 by calibration of the algorithm on various distributions. This allows keeping statistical noise to a constant level, as explained in [5].

Several zooms or rescaling are then performed in order to maximize resolution. The Twiss parameters (for a phase-space distribution) or covariance matrix (for a general case) are calculated and the histogram is shifted to normalised coordinates in order to have a round beam. Another set of zooms and rescaling is then done before starting the so called "wheel" algorithm.

Around the centroid of the beam, cross sections of the beam with regular angles in normalised coordinates are performed. This would allow having iso-density curves perpendicular to the cross sections in case of elliptically symmetric beams. The length of the cross section on each side of the centre is then adjusted to maximise resolution. The interpolated local densities are converted to a one-dimensional histogram with the same number of bins as

the two-dimensional one in either direction. The one dimensional algorithm is then applied to the cross section to determine a couple of points of the core-halo contour. The algorithm is explained in Fig. 1.

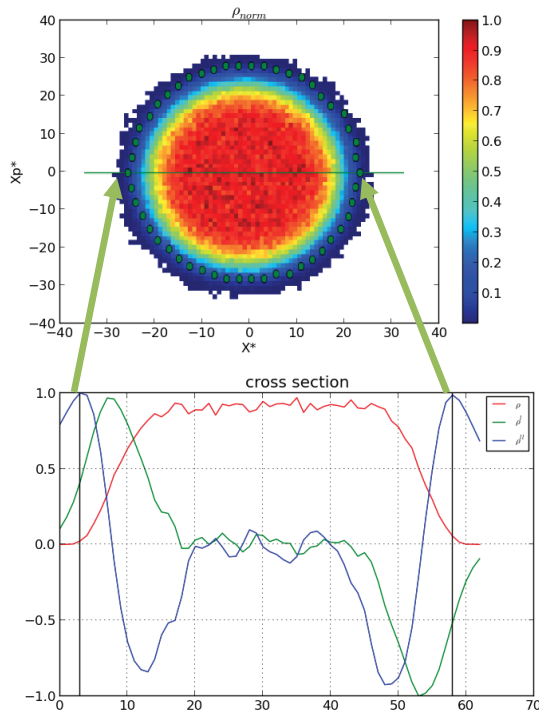


Figure 1: “Wheel” algorithm for finding the core-halo limit contour points in the normalized phase-space, the density and its derivatives from the cross section highlighted in green are shown below, as well as contour points.

The number of cross sections is determined so as to use as much information from the histogram as possible i.e. to have a radius going through each histogram bin halfway from the centre to the margin:

$$NbSections = \frac{\pi}{2\sqrt{2}} NbBins .$$

The points of the core-halo contour are then converted to real/phase-space and an algorithm based on the Jordan Curve Theorem determines whether the particles are inside of the core or part of the halo.

CHARACTERIZING THE TWO DIMENSIONAL HALO

Once the particles are split into core and halo, two concentration ellipses can be determined for each of them, from which emittance and Twiss parameters can be deduced for the core and for the halo separately.

The halo can be more precisely characterized in size and proportion by the two parameters PHP, percentage of halo particles and PHS, percentage of halo (phase-space) surface.

$$PHP = 100 \frac{Halo\ Particles}{Total\ Number\ of\ Particles}$$

$$PHS = 100 \frac{Halo\ Bins}{Total\ Number\ of\ Bins}$$

All these parameters, emittances, Twiss parameters, PHP and PHS can be used to characterize in more details a given beam distribution. They present the advantage of being usable as constraints for accelerator tuning in sight of halo or core minimization.

RESULTS

The algorithm described above was first applied to three typical distributions shown in the left part of Fig. 2. The first two look very different, one is rather bulky with very little halo and the other is very sharp with a big halo. They have actually identical global Twiss parameters and emittances: $\alpha=0.12$, $\beta=1.48$ (mm/ π .mrad), $\gamma=0.69$ (mrad/ π .mm), $\epsilon=3.97$ (π .mm.mrad). The third one is the simulated xx' phase-space distribution of the IFMIF-Lipac Deuteron beam after the superconducting HWR linac.

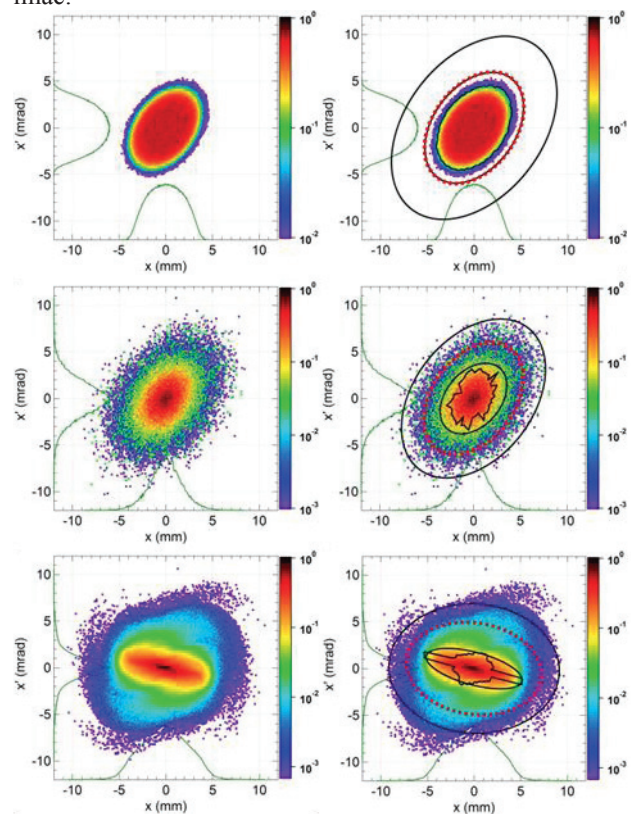


Figure 2: Three phase-space distributions and corresponding output from the algorithm, the first two have identical Twiss parameters.

The algorithms output is added to these distributions in the right part of Fig. 2: the 3 σ -Emittance ellipse for the whole beam, in dashed red, then for the core and halo and the irregular core-halo limit contour, in black. The corresponding output parameters are displayed in Table 1.

Table 1: Output Parameters for the Three Distributions, Emittances are Expressed in π .mm.mrad

Distribution	Little Halo	Big Halo	Linac Output
PHP	1.06%	37.5%	45.9%
PHS	23.2%	91.1%	95.6%
Whole emittance	3.3	3.3	3.96
Core emittance	3.24	1.2	1.03
Halo emittance	8.97	6.78	7.01

As it could be expected, the sharper distribution has more halo than the other with identical Twiss parameters. The core and halo emittances, the halo particles being around the core ones are respectively lower and higher than the whole emittance.

This algorithm was then applied to phase-space distributions all along of the IFMIF-Lipac resulting from transport simulations done with the Tracewin code [6].

Figure 3 shows the beam characterization from source extraction to final beam dump, using emittances of core and halos separately and PHP, PHS.

CONCLUSION

Strong self-forces in high intensity beams make beam halo play an important role. A precise determination of

the core-halo limit becomes necessary. This allows a precise determination of the core-halo limit to study core or halo growths instead of global emittance or other global RMS quantities. Instead of RMS beam envelope and the associated beam emittance, core and halo external limits along with halo particle number and halo surface can give a precise and more exhaustive view of the beam quality. The parameters defined here, emittances of the core or the halo, PHS, PHP could be used for minimizing more appropriately losses in high intensity accelerators.

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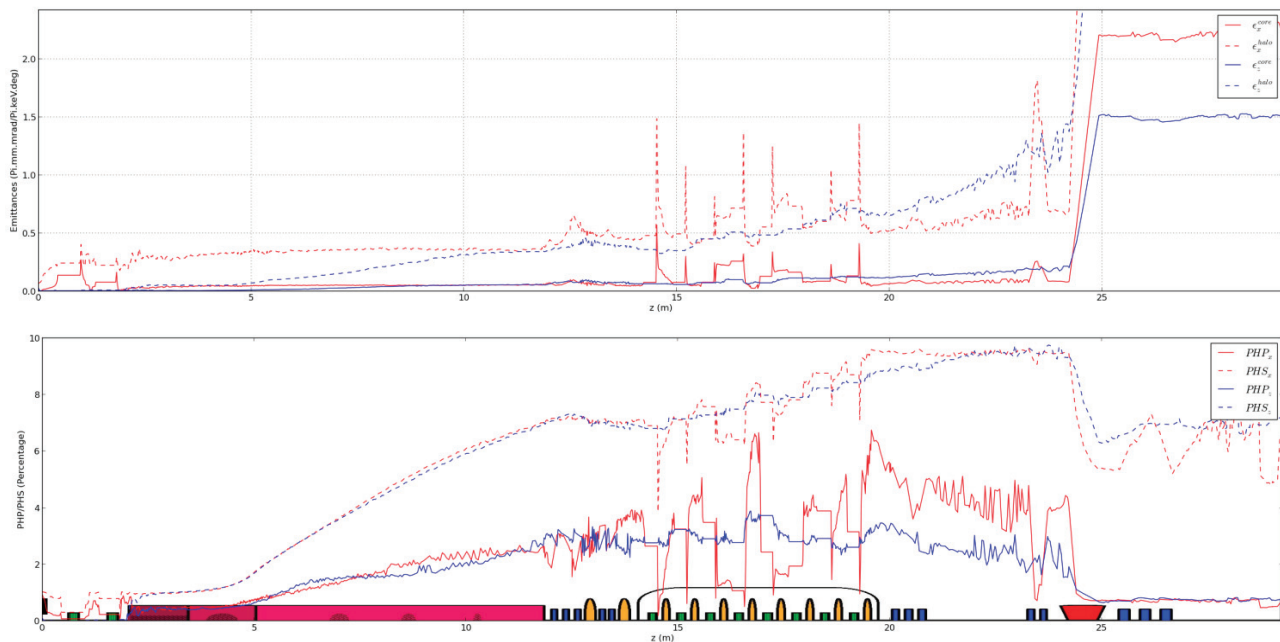


Figure 3: Beam characterisation in horizontal (x, red) and longitudinal (z, blue) along the IFMIF-LIPAc accelerator, from source extraction to final beam dump. Using emittances of the core and halo (top) and PHP/PHS (bottom).