Abstract
The proposed Advanced Photon Source (APS) multibend achromat (MBA) lattice includes a passive bunch-lengthening cavity to alleviate lifetime and emittance concerns. Feedback in the main radio-frequency (rf) system affects the overall impedance presented to the beam in this double rf system. To aid beam stability studies, a realistic model of rf feedback has been developed and implemented in elegant and Pelégenant.

INTRODUCTION
Many existing and planned storage ring light sources employ harmonic bunch-lengthening cavities [1–3]. For the required tuning in bunch-lengthening mode, the harmonic cavity contributes to Robinson mode growth which opposes the damping provided by the main rf cavities [4]. RF feedback on the main rf modifies the apparent impedance to the beam and hence influences Robinson instabilities [5]. Direct rf feedback can easily be accounted for by simply reducing the cavity shunt impedance and quality factor.

However, rf feedback systems rarely consist of, nor always include, pure direct rf feedback. They can work in either amplitude and phase (amp/phase) polar coordinates or in-phase and quadrature (I/Q) cartesian coordinates. They can also include integral feedback and other frequency shaping effects and need not be implemented symmetrically.

Including realistic rf feedback in particle tracking is essential to capture the interaction of the beam with the rf system as depicted in Fig. 1. Thus, a flexible framework for including rf feedback in elegant [6] and Pelégenant [7,8] has been developed. This has allowed rf feedback to be included in APS upgrade tracking studies [9].

SYSTEM MODEL
Figure 2 represents the rf feedback model added to elegant. The interaction between the beam current and the rf cavity is still modeled using elegant’s RFMODE element. However, instead of using a fixed generator voltage, the generator voltage is determined by amp/phase rf feedback. I/Q and combination feedback as shown are planned but not yet implemented.

The rf system signals are represented at baseband using a time-varying complex-envelope. For example, the total cavity voltage is $v_{cav}(t) = \Re\left\{ V_{cav}(t)e^{j\omega_{rf}t}\right\}$ where $j = \sqrt{-1}$, $\omega_{rf}$ is the rf drive frequency, and $V_{cav}(t) = V_I(t) + jV_Q(t)$ is the complex envelope with I/Q components $V_I(t)$ and $V_Q(t)$ respectively. The amp/phase are $|V_{cav}(t)| = \sqrt{V^2_I(t) + V^2_Q(t)}$ and $\angle V_{cav}(t) = \text{atan2}\left\{ V_Q(t), V_I(t)\right\}$.

Filters are provided to separately process the amplitude and phase errors with respect to reference setpoints. The outputs of these filters affect the generator current which drives a cavity I/Q state-space model to obtain the generator induced voltage. To facilitate initial conditions or feed-forward, a nominal generator current $I_G$, is provided.

Feedback Filters
The feedback filters are implemented as difference equations of the form

$$y[n] = \frac{1}{b_0} \sum_{i=1}^{r} b_i y[n-i] + \frac{1}{b_0} \sum_{i=0}^{m} a_i x[n-i]$$

where $y[n]$, $x[n]$ are respectively the filter output and input at sample $n$, and $y[n-i]$, $x[n-i]$ are past outputs and inputs from time $(n-i)T$ where $T$ is the sample period equal to an integer number of rf buckets. Four parallel filter blocks, each with an unlimited number of $a_i$ and $b_i$ filter coefficients can be supplied to elegant for both the amplitude and phase feedback.

Various methods exist to transform an analog filter (like that used at APS) to a digital one [10]. The bilinear transform produces a good match for an integrator.

Generator Induced Cavity Voltage
$I/Q$ modulation of the drive current, $I_G(t) = I_{f}(t)\cos\omega_{rf}t - I_{q}(t)\sin\omega_{rf}t$, produces $I/Q$ modulation of the cavity voltage. In general, the I/Q components of $V_{cav}$ will each contain a response to both I/Q components of $I_G$. The cavity state-space equations [11–13], when discretized using the zero-order hold method [10], become

$$\begin{bmatrix} V_I(n) \\ V_Q(n) \end{bmatrix} = \mathcal{A} \begin{bmatrix} V_I(n-1) \\ V_Q(n-1) \end{bmatrix} + \mathcal{B} \begin{bmatrix} I_{f}(n-1) \\ I_{q}(n-1) \end{bmatrix}$$

The matrices $\mathcal{A}$ and $\mathcal{B}$ are given as

$$\mathcal{A} = e^{-\sigma T} \begin{bmatrix} \cos\Delta\omega T & -\sin\Delta\omega T \\ \sin\Delta\omega T & \cos\Delta\omega T \end{bmatrix}$$

where $\sigma$ is the cavity I/Q state-space model to obtain the generator induced voltage. To facilitate initial conditions or feed-forward, a nominal generator current $I_G$, is provided.

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Figure 2: Block diagram of rf feedback model in elegant. I/Q feedback as shown is planned but not yet implemented.

\[ B = \frac{k N_{cav}}{\sigma^2 + \Delta \omega^2} \left[ \begin{array}{cc} \alpha & \beta \\ -\beta & \alpha \end{array} \right] \]  \hspace{1cm} (4)

where \( \Delta \omega = \omega_o - \omega_{rf} \) is the cavity detuning with \( \omega_o \) the cavity resonant frequency and \( \omega_{rf} \) the rf drive frequency, \( \sigma = \frac{\omega_o}{Q_L} \) is the cavity’s natural decay rate with loaded quality factor \( Q_L \), \( k = \omega_o \left( \frac{R}{Q} \right)_a \) is the single cavity loss factor with \( \left( \frac{R}{Q} \right)_a \) being the accelerator definition, and \( \alpha \) and \( \beta \) are

\[ \alpha = \Delta \omega e^{-\sigma T} \sin \Delta \omega T - \sigma e^{-\sigma T} \cos \Delta \omega T + \sigma \]  \hspace{1cm} (5)

\[ \beta = \sigma e^{-\sigma T} \sin \Delta \omega T + \Delta \omega e^{-\sigma T} \cos \Delta \omega T - \Delta \omega . \]  \hspace{1cm} (6)

The total number of cavities, \( N_{cav} \), is included in the definition of \( B \) in order to keep \( I_G \) relevant to a single cavity.

**APS STORAGE RING RF FEEDBACK**

Figure 3 represents the APS storage ring analog amp/phase rf feedback system. Two 1 MW klystrons would each drive six cavities. Only one system is shown. Due to operational considerations, the klystrons are operated with fixed rf drive while the modulating anode (mod-anode) is used to adjust the klystron output. Peak detectors monitor individual cavity voltages while a phase detector monitors the vector sum phase. Analog proportional-integral-derivative (PID) controllers are used for each loop.

The dynamic responses of all rf components were measured. Discrete-time equivalent models were then determined for use in elegant. To illustrate the effects of integral feedback and asymmetry, Fig. 4 shows calculated closed loop responses at resonance to small signal amplitude and phase fluctuations of the beam. The amplitude loop has lower bandwidth due to the slow response of the mod-anode. A direct rf feedback response with low gain of 1.5 is shown for comparison. Clearly integral feedback and asymmetry cannot be adequately modeled with simple direct rf feedback. Thus we felt it was prudent to include realistic feedback in tracking studies [9].

Figure 3: APS storage ring rf system.

**TRACKING WITH RF FEEDBACK**

Recent tracking simulations of the APS MBA lattice now include rf feedback [9]. Figure 5 shows feedback controlling the main rf generator forward power for 1 of 12 cavities to maintain the voltage during a balanced fill [9]. Due to proper input coupling, the reverse power decreases as beam...
is filled. Also shown is the beam-induced harmonic cavity voltage seen by each bunch. Figures 6-7, respectively, show the main rf system transients near the first and last bunch injections. The different response times of the amplitude and phase loops can clearly be identified near first injection. The large noise on the cavity voltage and phase is due to periodic beam loading from the unequal fill that exists until the last bunch is populated.

The transients seen at the last injection include modulations due to the beam-induced voltage fluctuating from damped synchrotron oscillations. The bunch centroids are shown in Fig. 8 for all bunches near the last injection, which occurs at pass 47,000. The variations across bunches seen between the second to last injection at pass 46,000 and the last injection are due to the ring being partially filled, resulting in periodic beam-loading of the main rf and each bunch seeing a slightly different synchronous phase.

Future studies will explore feedback alternatives such as comb-filters [12] for non-uniform bunch patterns and the effects of rf system noise.

CONCLUSION

Tracking simulations in elegant and Pelegant can now include realistic rf feedback. Flexible features include the ability to model various types of amp/phase feedback with independent multi-path filters. These capabilities are being used for APS upgrade tracking studies. Thus far no issues are foreseen with using our present rf feedback systems [9].

REFERENCES