Nonlinear Optics for Suppression of Halo Formation in Space Charge Dominated Beams

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Beam halo is a small fraction of particles (1% – 10%) which lies outside of the beam core and results in radio-activation and degradation of accelerator components.

Modern accelerator projects using high-intensity beams require keeping the beam losses at the level 1 Watt / m or less to avoid activation of the accelerator.

**Sources of Halo Formation in Linacs**

1. Mismatch of the beam with accelerator structure
2. Transverse-longitudinal coupling in RF field
3. Misalignments of accelerator channel components
4. Aberrations and nonlinearities of focusing elements
5. Beam energy tails from un-captured particles
6. Particle scattering on residual gas, intra-beam stripping
7. Non-linear space-charge forces of the beam
Injection of a continuous non-uniform beam in a focusing channel with linear field results in

(a) uniformity of beam core
(b) beam emittance growth
(c) halo formation

Example:
Beam energy 50 keV
Beam current 20 mA
Beam emittance $0.05 \pi \text{ cm mrad}$
FODO period 15 cm
Lens length 5 cm
Quadrupole field gradient 0.0428 T/cm
Tune depression $\sigma/\sigma_0 = 0.1$

(Numbers indicate focusing period)
Self-Consistent Beam Equilibrium in Focusing Channel

Self-consistent problem:

Vlasov’s Equation

\[ \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial P} \frac{dP}{dt} = 0 \]

Poisson’s Equation

\[ \Delta U_b = -\frac{\rho}{\varepsilon_0} \]


Example: Beam with Gaussian distribution function

\[ f = f_0 \exp\left(-2 \frac{x^2 + y^2}{R^2} - 2 \frac{p_x^2 + p_y^2}{p_\gamma^2}\right) \]

Total field \( E_{tot} = -\frac{mc^2}{q} \frac{1}{\gamma} \frac{\varepsilon^2}{R^4} r \)

Space-charge field

\[ E_b = -\frac{\partial U_b}{\partial r} = \frac{I}{2\pi \varepsilon_0 \beta c} \frac{1}{r} \left[ 1 - \exp\left(-2 \frac{r^2}{R^2}\right) \right] \]

Required focusing field

\[ E_{\text{ext}} = -\frac{mc^2}{q R \gamma} \left[ \frac{\varepsilon^2 r}{R^3} + 2 \frac{I}{I_c \beta \gamma} \frac{R}{r} \left( 1 - \exp\left(-2 \frac{r^2}{R^2}\right) \right) \right] \]
Quadrupole-Duodecapole Focusing Structure

Proposed four vane quadrupole structure with a duodecapole field component (EPAC96, p.1236)

Effective (time-independent) potential:

\[
U_{\text{eff}}(\vec{r}) = \frac{q}{4m\gamma} \frac{E^2(\vec{r})}{\omega^2}
\]

Potential of the uniform four vanes structure:

\[
U(r, \theta, t) = \left( \frac{G_2}{2} r^2 \cos 2\theta + \frac{G_6}{6} r^6 \cos 6\theta \right) \sin \omega t
\]

Lines of equal values of the function

\[
C = \frac{r^2}{2} + \zeta r^6 \cos 4\theta + \frac{\zeta^2}{2} r^{10}
\]

for \( \zeta = -0.03 \): (a) \( C = 0.05 \), (b) \( C = 0.25 \), (c) \( C = 0.5 \), and (d) \( C = 0.85 \)

\[
U_{\text{eff}}(r, \theta) = \frac{mc^2}{q} \frac{\sigma_0^2}{2} \left( \frac{r}{\lambda} \right)^2 \left[ 1 + 2\eta \left( \frac{r}{R} \right)^4 \cos 4\theta + \eta^2 \left( \frac{r}{R} \right)^8 \right]
\]

\[
\eta = \frac{G_6}{G_2} R^4
\]
Space-Charge Density of the Matched Beam


\[ U_b = -\frac{\gamma^2}{1 + \left(\frac{\beta \gamma I_c R^2}{2 I \epsilon^2}\right)} U_{\text{eff}} \]

Space charge density

\[ \rho_b = \rho_o \left(1 + 10 \xi r^4 \cos 4\theta + 25 \xi^2 r^8 \right) \]

Dynamics of 150 keV, 100 mA, 0.06 \( \pi \) cm mrad proton beam in a structure with \( G_2 = 48 \) kV/cm\(^2\) and \( G_6 = -1.3 \) kV/cm\(^6\).
Matched and Realistic Truncated Beam Distributions

(a) Self-consistent particle distribution \( \rho_b = \rho_o (1 + 10 \zeta r^4 \cos 4\theta + 25 \zeta^2 r^8) \) of the matched beam in quadruple-duodecapole channel with parameter \( \zeta = -0.03 \) and (b) beam with distribution \( \rho_b = \rho_o [1 - (r / R)^2]^2 \) truncated along equipotential lines of effective focusing field.
Dynamics of 150 keV, 100 mA, 0.06 $\pi$ cm mrad proton beam in a structure with $G_2 = 48 \text{ kV/cm}^2$.

Adiabatic matching of 150 keV, 100 mA, 0.06 $\pi$ cm mrad proton beam in a structure with $G_2 = 48 \text{ kV/cm}^2$, $G_6 = -1.9 \text{ kV/cm}^6$. 

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Combined FODO Structure with Arbitrary Multipoles*

Effective potential:

\[
U_{\text{eff}} = \left(\frac{\sigma_o \beta c}{L}\right)^2 \left[ \frac{r^2}{2} + f \zeta r^m \cos(m-2)\theta + \zeta^2 \frac{r^{2(m-1)}}{2} \right]
\]

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Figure 3: FODO quadrupole-duodecapole channel with combined lenses with the period of $L = 15$ cm, lens length of $D = 5$ cm, and adiabatic decline of duodecapole component to zero over a distance of 7 periods.

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Energy 35 keV
Current 11.7 mA
Emittance 0.05 π cm mrad
Quadrupole $G_2 = 0.03579$ T/cm

Quadrupole-Duodecapole Channel

Quadrupole Channel

Numbers indicate FODO periods
Fraction of particles outside the beam core \(2.5\sqrt{\langle x^2 \rangle} \times 2.5\sqrt{\langle y^2 \rangle}\) as a function of FODO periods: (blue) quadrupole channel, (red) quadrupole-duodecapole channel.
CST Particle Studio Simulation of Halo Formation in Quadrupole Channel
CST Particle Studio Simulation of Halo Suppression in Quadrupole-Duodecapole Channel
Final Particle Distributions in Focusing Channels

Quadrupole Channel

Quadrupole-Duodecapole Channel
Summary

1. Beam emittance growth and halo formation due to free-energy excess in high-brightness beams are unavoidable in linear focusing channel.

2. To prevent beam emittance growth and halo formation, focusing fields have to be a nonlinear function of radius.

3. Quadrupole-duodecapole focusing structure is an effective way to suppress beam halo formation.