INCOHERENT AND COHERENT EFFECTS OF SPACE CHARGE LIMITED ELECTRON CLOUDS

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Abstract
Recent studies show that the space charge limited (saturated) electron cloud generated by relativistic bunches has strongly inhomogeneous distribution. In particular, a dense electron sheath is formed near the pipe wall. This feature modifies the stopping powers and the microwave transmission compared with the uniform cloud case. In this paper we investigate further the influence of the space charge limited electron cloud on relativistic bunches. In particular, we focus on the incoherent tune spread and compare the results with the homogeneous pipe case. We derive analytical expressions governing the pinch dynamics of the saturated cloud in round geometry. The contribution of the electron cloud sheath to the wake fields is investigated as well. Findings of the analytical theory are then successfully compared with numerical particle-in-cell simulations.

INTRODUCTION

Electron clouds introduce diverse limitations on the performance of modern and future accelerators [1]. They lead to cryogenic heat load, pressure rise, emittance growth, synchronous phase shift and coherent beam instabilities. The last three effects depend on the wakefields induced by bunches in an electron cloud.

In the present paper we continue to study of how the saturated electron clouds affect relativistic proton bunches. In the first section we address the contribution of an electron cloud sheath to the longitudinal wakefields. Such kind of sheath was found to be formed at the saturation of the electron cloud buildup under the LHC conditions [3]. In the second section the studies of the tune shifts induced by electron clouds are presented. The possible influence on the electron cloud density measurements is also discussed.

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In Ref. [2, 3] it was pointed out that in some cases at saturation the electron cloud density at the pipe wall is significantly larger than the density in the pipe center, i.e. a dense electron cloud sheath is formed near the wall. It was shown that the wakefields induced by this distribution differ from the uniform cloud case. In this section we investigate the pinch of a ring-like electron cloud and the longitudinal wakefield induced by such an electron cloud distribution.

If the pinch dimension is much larger longitudinally than transversely, which is typically the case, then the electron cloud potential at the center is governed approximately by the local transverse charge distribution:

\[ \phi(z) \approx \frac{e \lambda_e}{2\pi \varepsilon_0} \ln \left( \frac{R(z)}{R_0} \right). \]

One can then obtain the longitudinal electric field of the ring-like charge distribution at the pipe center:

\[ E_z(z) = -\frac{\partial \phi(z)}{\partial z} = \frac{e \lambda_e}{2\pi \varepsilon_0} \frac{R'(z)}{R(z)}. \]

For Gaussian and rectangular bunch shapes this field can be calculated analytically.

![Figure 1](image_url)

Figure 1: (Color) Electric field of the ring-like electron cloud under the influence of the passing proton bunch. \( N_i = 2 \times 10^{11}, \sigma_z = 10 \text{ cm and } R_0 = 2 \text{ cm.} \)

Table 1 shows the analytical and numerical longitudinal wakefields on the pipe axis for two different bunch populations for the ring-like electron cloud distribution. One
Table 1: Simulation parameters for the LHC-like bunches.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch length, $\sigma_z$ [m]</td>
<td>0.1</td>
</tr>
<tr>
<td>Bunch radius, $\sigma_r$ [m]</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Bunch population, $N_i$</td>
<td>$2.5 \times 10^{10} - 6 \times 10^{11}$</td>
</tr>
<tr>
<td>Bunch spacing [ns]</td>
<td>25</td>
</tr>
<tr>
<td>Pipe radius, $R_p$ [m]</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>Magnetic field, B [T]</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum total SEY, $\delta_{max}$</td>
<td>1.4</td>
</tr>
<tr>
<td>Energy of $\delta_{max}$, $W_{SEY,max}$ [eV]</td>
<td>250</td>
</tr>
<tr>
<td>Reflection probability</td>
<td>1.0</td>
</tr>
<tr>
<td>Rediffusion probability, $\delta_{rd}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Ring circumference, $C$ [m]</td>
<td>$2.7 \times 10^4$</td>
</tr>
<tr>
<td>Fraction of the ring with EC</td>
<td>0.1</td>
</tr>
<tr>
<td>Betatron tunes $Q_x/Q_y$</td>
<td>65.32/63.27</td>
</tr>
<tr>
<td>Lorentz factor, $\gamma_0$</td>
<td>450</td>
</tr>
</tbody>
</table>

can see that the simulations predict nearly the same field that, however, abruptly cancels behind the bunch due to the electron cloud loss at the wall. Integrating Eq. 5 over $R_0$ one can obtain a wakefield for a cloud with a finite thickness $d$ and density $\rho_e$. This is the subject of our future studies.

**INCOHERENT TUNE SHIFT DUE TO SATURATED ELECTRON CLOUD**

In this section we analyze the effect of the saturated electron cloud on the bunch tune distribution. For this purpose, firstly, we perform buildup simulations until the saturation of the electron cloud is reached. The buildup starts from the initially uniform electron cloud. No primary electrons are produced during the buildup. Secondly, we use the obtained electron cloud profiles to track the beam particles and calculate the corresponding tune shifts. In these calculation we neglect the synchrotron motion and assume a simple constant focusing lattice. The resulting tune shifts are compared with the uniform electron cloud models. Average or central densities of the realistic electron cloud are assigned to the uniform clouds. In all the simulations we study the simplified case of the circular geometry similar to the LHC beam pipe. The simulation parameters are listed in Table 1.

Fig. 2 shows the average and central electron cloud densities versus the bunch population. The curves are accompanied by the evolution of the heat load. The heat load is calculated for the saturated cloud as energy deposited on the wall during one bunch passage divided by the bunch spacing. In the studied case one can see that the average electron cloud density has a maximum around $1.5 \times 10^{11}$ particles per bunch. On the contrary, the central density reaches minimum at $3 \times 10^{11}$. At this point the cloud is mainly distributed into two vertical stripes. Further increase of intensity leads to the formation of another stripe in the middle of the pipe and, thus, the growing central density.

Fig. 4 and Fig. 5 show the average tune shift induced by the realistic and two chosen uniform clouds as a function of intensity. It is important to notice the $N_i = 3 \times 10^{11}$ case (See Fig. 3). When two dense stripes are formed, the tune shift becomes negative in one direction and positive in another direction. Both of the uniform cloud models give positive tune shifts. It is also worth noting that all the models give dissimilar dependence of the tune shift and the tune spread on the bunch population.

The measurements of the electron cloud density based on the tune shift typically utilize the following formula:

$$\bar{\rho} = \frac{\gamma_0}{r_e L} \left( \frac{\Delta Q_x}{\beta_x} + \frac{\Delta Q_y}{\beta_y} \right),$$

where $\bar{\rho}$ is the average electron cloud density along the bunch, $\gamma_0$ is the Lorentz factor, $r_e$ is the electron classical radius, $L$ is the circumference, $\Delta Q_x$ and $\Delta Q_y$ are the horizontal and vertical measured tune shifts, $\beta_x$ and $\beta_y$ are the average horizontal and vertical beta-functions. In case of two stripes the tune shift contributions in horizontal and vertical plane nearly cancel and give approximately zero density. This happens in agreement with the Poisson equation for zero charge density: $\partial E_x/\partial x = -\partial E_y/\partial y$. Similar electron distributions are very common in accelerators (See for example...
Ref. [5,6]). Thus, the measurements of the tune shift would mainly include only the information about the central density in field-free regions.

Fig. 6 shows the electron cloud densities averaged along the bunch and reconstructed using Eq. 6. One can observe that the dissimilarity between the realistic and diluted models almost disappears. The values are larger than given in Refs. [5,6].

**CONCLUSION AND OUTLOOK**

We have studied the transverse and longitudinal effects of the saturated electron clouds in circular pipe geometry. Using the kick approximation we obtained the longitudinal wakefields for the ring-like electron cloud distribution. The resulting analytical expressions were successfully compared with the simulations. In future these expressions can be extended for the case of a thick electron cloud sheath.

A phenomenological study of the incoherent tune shifts due to the saturated electron cloud was performed for a broad intensity range. We focused on the electron cloud in an external dipole magnetic field. In most of the studied cases a significant difference between the horizontal and vertical tune shifts depending on the model can be observed.

In case of the "two-stripe" distribution and in agreement with the Poisson equation, our simulations give the horizontal and vertical tune shifts of the opposite signs. In the extreme case, the contribution of such distribution to the density measurements can become zero. In the meantime, uniform cloud models always yield positive tune shifts.

However, when horizontal and vertical tune shifts are used to extract the electron cloud density, the diluted and realistic models give nearly the same results for \( N_i < 3 \times 10^{11} \). Under the studied conditions, the features of the realistic saturated electron cloud start to play role for \( N_i \geq 3 \times 10^{11} \). On the contrary, the average cloud model gives almost always incorrect results.

**ACKNOWLEDGMENT**

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**REFERENCES**