HOSING INSTABILITY SUPPRESSION IN SELF-MODULATED WAKES DRIVEN BY NON-UNIFORM DENSITY PROFILE PARTICLE BUNCHES

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Abstract

The self-modulated plasma wakefield accelerator uses long particle bunches (longer than the plasma wavelength) to excite large amplitude wakefields through the self-modulation instability or SMI. The hosing instability, or HI, is a competing process that can lead to bunch breakup before SMI occurs, thereby limiting the maximum energy that can be reached by an externally injected particle bunch. It has recently been shown analytically, however, that hosing can be suppressed after SMI saturation in the case of uniform density bunches. Here we present analytical and numerical solutions that demonstrate HI suppression after SMI saturation of drivers with non-uniform density profiles.

INTRODUCTION

In the pursuit of higher energy gain plasma accelerators reaching the energy frontier, CERN has recently approved a Proton Driven Plasma Wakefield Acceleration (PDPWFA) experiment [1] known as AWAKE [2]. The AWAKE experiment and current laser (LWFA) and plasma wakefield acceleration (PWFA) experiments operate in different regimes. The length of the driver in current LWFAs [3] and PWFAs [4] is typically on the order of the plasma wavelength (λ_p), thereby maximising the amplitude of the initial wakefield. The AWAKE experiment, however, will use 10 cm long proton bunches, much longer than λ_p, to drive plasma waves even at plasma densities as low as $n_0 \simeq 10^{14} - 10^{15}$ cm^{-3}. The excitation of large amplitude wakefields in experiments at CERN will then occur through the SMI [5]. Self-modulated PDPWFA experiments will take place during in 2-4 years. The physics of SMI, however, is being currently tested under similar conditions at SLAC in the E-209 experiment [6, 7]. The E-209 experiment uses the uncompressed 500 μm long electron and positron bunches propagating in plasmas with $n_0 \simeq 10^{16} - 10^{17}$ cm^{-3}.

The SMI is an instability that leads to radial modulations of the bunch density profile. This modulation, which occur roughly at the plasma wavelength [5], grows along the bunch for longer propagation distances. After SMI saturation, the initially long bunch becomes self-modulated into a train of smaller bunches, each smaller than λ_p, that are separated by the plasma wavelength (λ_p). The resulting self-modulated bunch train can resonantly excite wakefields that grow secularly along the bunch. Although the wake phase velocity can vary significantly during its linear stage [8], it stabilises and becomes closer to the driver velocity after SMI saturation [9]. Ideal conditions for particle acceleration are then met once the bunch becomes fully self-modulated.

The HI is a competing instability [10], which consists of unstable oscillations of the bunch centroid. The HI arises from the mismatch between the centroids of the bunch and plasma wave, and can lead to bunch breakup before self-modulation saturates [11]. The occurrence of hosing may then limit the acceleration distance to the growth length of HI, thereby strongly limiting the maximum energies that could be reached in self-modulated wakefield accelerators.

It has been recently shown, however, that hosing can be suppressed after the saturation of SMI if the wake excitation is in the linear PWFA regime [12]. Reference [12] shows that the centroid of each self-modulated beamlet executes harmonic oscillations that are driven by the preceding beamlet oscillations. Thus, HI suppression can occur when the frequency of oscillation of each centroid differs, preventing resonant amplification of centroids oscillations. This condition is automatically satisfied in the linear regime, where the wake amplitude, and then the centroid frequency of oscillation, grow along a fully self-modulated bunch.

So far, the suppression of the HI after the saturation of SMI was examined analytically considering flat-top bunches. More realistic bunch profiles, such as those that will be used in experiments, however, have non-uniform density profiles with Gaussian-like shapes. SMI seeding, however, requires bunches with sharp rise times, see Refs. [6, 13, 14], which can be approximated by a triangular density profile for calculations. The wakefields driven by triangular or flat-top bunches differ. Exploring the suppression of the hosing instability for a train of self-modulated pulses generated from a bunch with a triangular density profile is therefore interesting for experiments. In these proceedings we will then show analytically and numerically that hosing instability can also be suppressed after SMI saturation for bunches with triangular profile.

HOSING SUPPRESSION IN DRIVERS WITH UNIFORM DENSITY PROFILES

We start by analysing the equation for the centroid evolution of a fully self-modulated train of bunches. To understand the main physical mechanisms leading to hosing suppression in this configuration we consider a simplified discrete particle model where the density of each self-modulated beamlet is represented by $n_{\parallel} \propto \delta\left(\xi - \xi_p\right)$, where $\delta(x)$ is the Dirac delta function, $\xi = z - v_b t$, z is the longitudinal position, v_b the speed of bunch, t the time, and

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$\xi_p = z_p - v_b t$, where z_p is the beamlet longitudinal position. The density profile of the fully self-modulated train of m beamlets is thus given by $n_{\parallel}^{\mathrm{SM}} = \sum_{l=0}^{m} k_p^{-1} n_l \delta(\xi - \xi_l)$, where ξ_l is the location in ξ of the l^{th} beamlet, and where $k_p^{-1} n_l$ is proportional to the total charge of the l^{th} beamlet. The evolution of the centroid of each beamlet is thus given by [12]:

$$\frac{\mathrm{d}^2 x_m}{\mathrm{d}z^2} + x_m(z)\frac{q_b^2 k_\beta^2}{e^2}\left(\frac{\delta n}{n_0}\right)_m = k_\beta^2 \sum_{l=0}^{m} n_l w_l x_l(z), \quad (1)$$

where $x_m = x_c(z,\xi_m)$, $(\delta n/n_0)_m = \sum_{l=0}^{m} n_l \sin\left[k_p(\xi_l - \xi_m)\right]$ is the amplitude of the plasma density fluctuations at $\xi = \xi_m$, and $w_l = \sin\left[k_p(\xi_l - \xi_m)\right]$ is a weighting factor depending on the relative position between beamlets. Equation (1) is that of an harmonic oscillator where each centroid performs transverse oscillations at $k_m^2 = k_\beta^2(\delta n/n_0)_m$. In addition, each m^{th} centroid oscillation is also driven by the weighted centroid oscillations of preceding beamlets given by $\sum_{l=0}^{m-1} n_l w_l x_l$.

Although exact analytical solutions of Eq. (1) can be retrieved for a small number of beamlets, generalised results for an arbitrary number of beamlets are not available. Thus, in order to determine analytical expressions for the evolution of x_m in self-modulated regimes we simplify Eq. (1) considering constant $w_l = \alpha$ and $k_m^2 = \beta k_\beta^2 \sum_l n_l$. Since simulations show that α and β are similar and roughly constant along the bunch, and since in this case the hosing suppression mechanisms are independent of the exact values of α or β, we take $\alpha = \beta = 1$. Replacing the sums by integrals in Eq. (1) yields [12]:

$$\left(\frac{1}{k_p k_\beta^2}\frac{\mathrm{d}^2}{\mathrm{d}z^2} + \int_{\infty}^{\bar{\xi}} n_{\parallel}(\xi')\mathrm{d}\xi'\right) x_c = \int_{-\infty}^{\bar{\xi}} x_c(\xi') n_{\parallel}(\xi')\mathrm{d}\xi', \quad (2)$$

where $\bar{\xi} = \xi\sigma_z/\Delta\xi$, $\Delta\xi$ is the separation between beamlets, $k_\beta^2 = k_p^2 m_e n_{b0}/(2\gamma m_b n_0)$ and $k_p = \sqrt{4\pi n_0 e^2/m_e}$ are the betatron and plasma wavenumbers, e and m_e the electron charge and mass, m_b and γ the bunch particles mass and relativistic factor.

Equation (2) gives $x_c(z,\xi)$ for fully-self modulated bunches and arbitrary $n_{\parallel}$. Flat ($n_{\parallel} = 1$) and linearly rising density ($n_{\parallel} = \eta\xi$, where $\eta > 0$) bunches yield straightforward analytical expressions for x_c. The solution to Eq. (2) for $n_{\parallel} = 1$, with initial conditions $x_{c0} = \delta_{\mathrm{HI}}\xi$ and $\mathrm{d}x_{c0}/\mathrm{d}z = 0$ is:

$$k_p x_c = \frac{2\delta_{\mathrm{HI}}}{k_\beta^2 z^2}\left[-1 + \cos(N_{\mathrm{flat}}) + N_{\mathrm{flat}}\sin(N_{\mathrm{flat}})\right], \quad (3)$$

where $N_{\mathrm{flat}} = k_\beta z\sqrt{k_p\xi}$, and $\delta_{\mathrm{HI}}\xi$ is the initial bunch tilt that seeds HI. Equation (3) indicates that $k_p x_c \propto \delta_{\mathrm{HI}}\sqrt{k_p\xi}/(k_\beta z)$ clearly demonstrating HI damping after SMI saturation. Although x_c still increases along the bunch, centroid oscillations decrease for larger z. Similar conclusions hold for triangular bunches with $n_{\parallel} = \eta k_p \xi$ ($\eta > 0$) for which:

$$k_p x_c = \sqrt{\frac{2}{\eta}\frac{\delta_{\mathrm{HI}}}{k_\beta z}}\sin\left(k_\beta z k_p \xi\sqrt{\frac{\alpha}{2}}\right). \quad (4)$$

Equation (4) shows that the amplitudes of the centroids oscillations are constant along the bunch, being damped for longer propagation distances.

Comparison between the analytical predictions for fully self-modulated beams [Eqs. (3) and Eq.(4)] with numerical solution of Eq. (2) are shown in Fig. 1. Figure 1a shows the evolution of centroid oscillations at different propagation distances for a flat-top bunch. The initial bunch displacement is given by the dashed line. Figure 1b compares the numerical solution of Eq. (2) for the centroid oscillations of a triangular bunch with $\eta > 0$, with corresponding analytical results given by Eq. (4). Since Eqs. (3) and (4) are exact solutions of Eq. (2), the numerical solutions are indistinguishable from analytical results. Analytical results for $\eta < 0$ are not available, but numerical solutions of Eq. (2) are shown in Fig. 1b, also indicate that hosing is mitigated in this case.

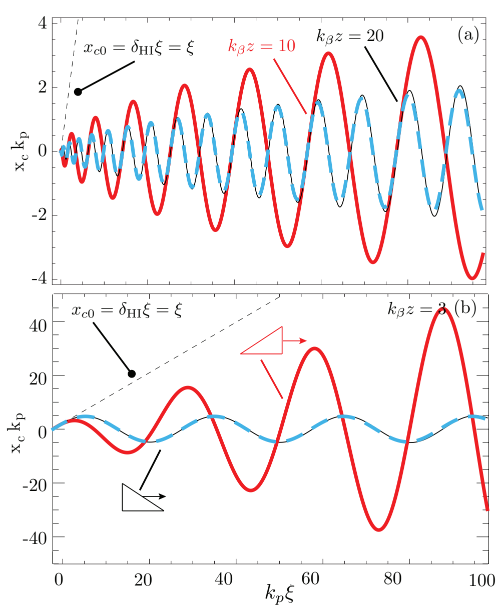

Figure 1: Numerical (solid lines) and theoretical (dashed lines) beam centroid displacement (x_c) for a self-modulated bunch. (a) x_c for a fully self-modulated flat-top bunch for propagation distances $k_\beta z = 10$ (numerical - thick red solid line) and $k_\beta z = 20$ (theory - thin solid line ; numerical - thick blue dashed line) (b) x_c for a fully self-modulated bunch with a linear rise (theory - thin black line; numerical - thick blue dashed line) and fall (numerical - thick red line) current profile. Thin dashed lines indicate the initial centroid displacement that seeds the hosing instability.

The bunch profile with $\eta < 0$ corresponds to a bunch with a sharp rise time, required, for instance, to seed SMI, and it is therefore important for experiments. As stated in the introduction, the amplitude of the wakefield saturates towards the back of the bunch in this case. This is illustrated in Fig. 2. As a result, the frequency of the oscillations of each centroid becomes identical at the back of the bunch. This could lead to HI growth because centroid oscillations would be ressonantly driven by the preceding beamlets at the back of the bunch. However, Eq. (1) also indicates that the weight of the driving oscillations associated with lower density beamlets is also lower. The result shown in Fig. 1 then shows that the latter effect dominates over the former.

It is also interesting to compare the case with $\eta > 0$, i.e. bunch with a long rise time, with the case with $\eta < 0$, i.e. bunch with a sharp rise time, both shown in Fig. 1b. For the case with $\eta > 0$ (long rise time) the wakefield grows more rapidly along the bunch than the wake excited by bunches with short rise times (Fig. 2). It is therefore more difficult to meet the resonance condition when $\eta > 0$ than when $\eta < 0$, and, as a result, the amplitude of the centroid oscillations are also lower when $\eta > 0$, as shown in Fig. 1b. However, SMI will not be seeded as effectivelly when the bunch has a long rise time. Thus, although after SMI saturation, profiles with long rise times lead to lower centroid oscillations in comparison to bunches with short rise times, it is less likely that bunches with $\eta > 0$ can reach SMI saturation (unless SMI is seeded by any other means).

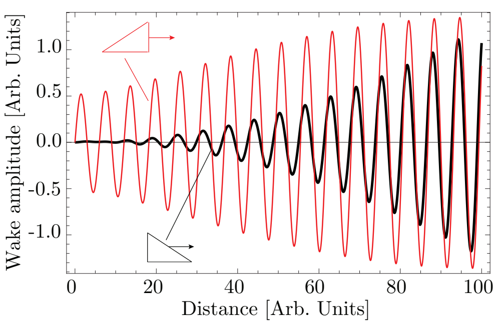

Figure 2: Wakefield amplitude for a bunch with a sharp rise time (thin red line) and for a bunch with a long rise time (thick black line). The density profile of the bunch with the short rise time is $n_{\|} \propto (L - \xi)\,[1 + \delta \cos(\xi)]$ and for the bunch with the long rise time is $n_{\|} \propto \xi\,[1 + \delta \cos(\xi)]$ where L corresponds to the bunch length, and $\delta \ll 1$ a parameter that controls the amount of bunch self-modulation.

CONCLUSIONS

In conclusion, we have showed that hosing suppression in self-modulated plasma wakefields can occur in the linear regime for flat density bunches and for bunches with triangular density profiles. These analytical results then confirm simulation results presented in Ref. [12] in that the propagation of realistic bunches with non-flat density profiles can be hosing free provided that self-modulation is seeded such that it can reach saturation before hosing saturates.

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