DESIGN AND SIMULATION OF A HIGH FIELD - LOW ENERGY MUON IONIZATION COOLING CHANNEL *

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Abstract

Muon beams are generated with inherited large transverse and longitudinal emittances. In order to achieve low emittance within the short lifetime of the muons, the only feasible cooling scheme is the ionization cooling. In this study we present a design and simulation of a novel ionization cooling channel. The channel operates at a very strong magnetic fields of 25-30 T with low muon beam energy starting from 66 MeV and decreasing gradually. We study the beam dynamics of such low energy beam in high field region inside and between cooling stages. Key design parameters will be presented and in addition the performance and channel requirements of RF cavities and high field magnets will be presented.

INTRODUCTION

Muon accelerators are required to deliver high luminosity muon beams with transverse emittances less than 50 μm-rad for muon collider rings. Reduction of the initial large 6D transverse emittance of muon beam is achieved only by ionization cooling, where muons suffer momenta reduction in all directions as they pass through any absorber material. Absorbers are usually followed by a series of RF cavities to restore only the longitudinal component of the momentum. This will only cool the transverse emittance, and if combined with emittance exchange it can reduce the 6D phase space. 6D ionization cooling channels operate with muon beams of central momenta ≈220 MeV/c and can not provide cooling beyond $\epsilon_T \approx 280 \mu m \cdot rad$ and $\epsilon_L \approx 1.5 mm$ [1–3].

We are presenting a design and simulation study of a novel ionization cooling channel that operates under very strong focusing fields of 25-30 T in a much lower momentum regime of 135 MeV/c that decreases gradually through the channel. At such low momenta, the longitudinal emittance rises from the negative slope of energy loss versus energy. Starting with initial emittances achieved by 6D cooling channels, we show that the presented channel design can achieve transverse emittance of 55 μm-rad within the longitudinal acceptance of 76 mm-rad. This allows using this channel design in the Muon collider final cooling stages.

Inside the absorber material, the reduction in beam momenta providing transverse cooling is opposed by the multiple Coulomb scatterings. The multiple Coulomb scatterings in the absorber material limits the ionization cooling and act as a heating element. Strong focusing fields at the absorber location helps in minimizing this heating effect. Equ. 1 gives the rate of change of the normalized transverse emittance inside an absorber material

$$\frac{d\epsilon_N}{ds} = -\frac{\epsilon_N}{\beta^2 E} \frac{dE}{ds} + \frac{\beta_L E_s^2}{2 \beta m c^2 L_R E}$$  \hspace{1cm} (1)

Where $\beta_L (\approx 2 P/0.3 B)$ is the transverse betatron function of the beam, $\beta = v/c$, $L_R$ is the radiation length of the absorber material, and $E_s = 13.6$ MeV.

When both the ionization cooling and the multiple scattering heating in the absorber material come to equilibrium, the normalized transverse emittance of the beam can not be reduced beyond the value of the equilibrium emittance given by

$$\epsilon_{equ.N} = \frac{\beta_L E_s^2}{2 \beta m c^2 L_R (dE/ds)}.$$  \hspace{1cm} (2)

Which gives the minimum achievable emittance in any absorber material in a simpler form

$$\epsilon_{min.N} \propto \frac{E}{BL_R (dE/ds)}.$$  \hspace{1cm} (3)

For a given absorber material $\epsilon_{min.N}$ depends on the beam energy $E$ and the focusing strength $B$. Reducing the beam energy gradually in the cooling channel with strong focusing fields can provide a reduction in transverse emittance to values in the range of 60-25 μm-rad. Muon ionization cooling to emittance below 50μm-rad, can be achieved with liquid hydrogen absorbers placed in high field solenoids which has small bores.

Figure 1: Ionization energy loss of muon beam inside different absorber materials.

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HIGH FIELD MAGNET

A set of eight coaxial superconducting solenoid coils were designed to deliver peak fields up to 50 T. The coils parameters and currents are given in Table 1 and schematic of such magnet is shown in Fig. 2. In order to provide peak fields ranging from 30 to 25 T, the current densities are scaled accordingly.

Table 1: High field Magnet Parameters

<table>
<thead>
<tr>
<th>Length [m]</th>
<th>Inner radius [m]</th>
<th>Thickness [m]</th>
<th>I/A [A/mm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.317</td>
<td>0.025</td>
<td>0.029</td>
<td>164.26</td>
</tr>
<tr>
<td>0.337</td>
<td>0.055</td>
<td>0.041</td>
<td>142.43</td>
</tr>
<tr>
<td>0.375</td>
<td>0.098</td>
<td>0.056</td>
<td>125.88</td>
</tr>
<tr>
<td>0.433</td>
<td>0.157</td>
<td>0.067</td>
<td>119.07</td>
</tr>
<tr>
<td>0.503</td>
<td>0.228</td>
<td>0.120</td>
<td>85.99</td>
</tr>
<tr>
<td>0.869</td>
<td>0.355</td>
<td>0.089</td>
<td>39.60</td>
</tr>
<tr>
<td>0.868</td>
<td>0.454</td>
<td>0.104</td>
<td>44.30</td>
</tr>
<tr>
<td>0.992</td>
<td>0.575</td>
<td>0.252</td>
<td>38.60</td>
</tr>
</tbody>
</table>

LATTICE DESIGN

The high-field low energy cooling channel is composed of 17 stages each stage is composed of a liquid hydrogen absorber placed inside the innermost coil where \( \beta_L \) has a minimum value. Each focusing solenoid is preceded and followed by a set of matching coils to match the beam transported in a 3.5 T constant focusing field to the peak 30-25 T value. Due to the fact that the beam mean momentum before cooling is quite different from that after cooling, the matching coils are asymmetric. A drift space follows the absorber in order to develop an energy-time correlation required for the energy-phase rotation. The first set of RF cavities after the drift are set to have 0 phase in order to be able to rotate the longitudinal phase space distribution to upright position. Another set of RF cavities following the phase rotation section are used to accelerate the muon beam to the required energy for the following cooling stage. A one stage layout is given in Fig. 3. In addition to this a set of field flip matches are inserted between some of the stages to limit the accumulation of the canonical angular momentum.

Because the beams energy is reduced gradually and the bunch length is increasing gradually, each stage has its own set of matching coils, focusing coils, RF cavities which are different from any other stage. Figure 4 shows the structure of the first three stages.

In the first stage a set of 325 MHz RF cavities are used and are placed inside the 3.5 T transport coils. In the later stages as the bunch gets longer which mandates the use of lower RF frequencies, the size of the RF cavities exceeds the size of the transport coils, and the cell structure switches to a smaller coils within the RF field. Simulation study of the channel performance was carried using G4Beamline. Initial muon beam emittances used in the simulation are \( \epsilon_T \approx 280 \mu \text{m-rad} \) and \( \epsilon_L \approx 1.5 \text{ mm} \).

Table 2: Absorbers and RF Cavities Parameters

<table>
<thead>
<tr>
<th>Stage</th>
<th>Absorber Length [cm]</th>
<th>RF freq. [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>65-60</td>
<td>325-201</td>
</tr>
<tr>
<td>6-10</td>
<td>55 - 35</td>
<td>201-100</td>
</tr>
<tr>
<td>11-15</td>
<td>25 - 13</td>
<td>100 - 70</td>
</tr>
<tr>
<td>16 -17</td>
<td>12-10</td>
<td>60 - 20</td>
</tr>
</tbody>
</table>

Figure 2: A set of eight superconducting coaxial coils providing a peak field of 50 T. The inner radius of the smallest coils is 0.025 m.

Figure 3: Schematic of the elements of one ionization cooling stage.

Figure 4: First three stages showing the absorbers (blue), drift (gray), and RF cavities (red). The focusing coils are made transparent.
COOLING CHANNEL PERFORMANCE

The initial energy, energy spread, and absorber length for every stage are optimized to minimize $-\frac{d\epsilon}{d\epsilon}$. At the initial stages, to avoid the emittance growth from the amplitude dependent transit time, the field integral is minimized to provide just the required focusing for reducing the $\epsilon_{eq}$ with a short decline to the transport field value. The transverse longitudinal coupling for large amplitude particles at the early stages may lead to unnecessary increase in the bunch length, the difference between the timing of a zero amplitude particle and a second particle with non-vanishing amplitude is given by

$$\Delta t \approx \frac{p^2 E}{2c^2 p^3} \int_0^Z \frac{B_z(z')}{B_z(0)} dz'$$  \hspace{1cm} (4)

As the energy of the beam is further decreased, the growth of the longitudinal emittance is increased and the cooling will require more stronger field of 30 T.

A set of liquid hydrogen absorbers were used in the simulations. The absorbers length range from 65 to 10 cm. As the bunch goes through the energy phase rotation section in each stage its get longer, we start the simulation of the channel with $\sigma_t = 5$ cm and the final bunch length is 1.8 m.

The RF frequencies were chosen to keep the bunch length $\sigma_{ct} < \lambda/20$. Table 2 shows RF parameters used in every stage. The gradients assumed maximum surface fields $\approx \sqrt{f}$, and assuming reentrant vacuum cavities with surface to accelerating gradients $\propto f^{0.75}$. Only the last accelerating stage required an induction linac.

CONCLUSION

A complete design and simulation of a high field - low energy cooling channel was presented. The peak field considered in this study was limited to within the current state of the art magnets with 30 T. The design parameters of the cooling channel was discussed. Future work will include global optimization of the channel to improve cooling and transmission.

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REFERENCES

