ENERGY-SILENCED HGHG*  
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Abstract

We study the effect of longitudinal space charge on the correlated energy spread of a relativistic beam that has been microbunched for the emission of high harmonic radiation. We show that, in the case of microbunching induced by a laser modulator followed by a dispersive chicane, longitudinal space charge forces can act to significantly reduce the induced energy spread of the beam without a reduction in the harmonic bunching content. This effect may significantly relax constraints on the harmonic number achievable in HGHG FELs, which are otherwise limited by the induced energy spread from the laser.

INTRODUCTION

Free-electron lasers (FELs) use relativistic electron beams to produce widely tunable light with exceptional brightness at wavelengths down to hard x-rays for a broad range of studies. To improve the FEL performance and temporal coherence, high-gain harmonic generation (HGHG) is a technique in which the electron beam is first modulated in energy by a laser, and then density modulated in dispersive transport [1–3]. In this process, electrons are piled into sharp peaks longitudinally that are spaced at the laser wavelength, and at harmonics $h$. The bunching factor for a beam with an initially uncorrelated Gaussian energy spread $\sigma_E$ is:

$$b_h = e^{-(hB)^2/2} J_h(-hAB),$$  \hspace{1cm} (1)

where $A = \delta\eta/\sigma_{\eta 0}$ is the laser energy modulation amplitude relative to the relative energy spread $\sigma_{\eta 0} = \sigma_E/E$, $B = kR_{sd}\sigma_{\eta 0}$ is the scaled dispersion, and $J_h$ is the Bessel function of order $h$.

The harmonic number in HGHG FELs is typically limited to $h \sim \rho/\sigma_{\eta 0} \sim 10$ in modern machines, where $\rho$ is the FEL frequency bandwidth at saturation [4]. From Eq. (1) the optimal energy modulation to obtain significant bunching at the frequency $hk$ is $A = 1/B \approx h$. However, the FEL saturates when the e-beam energy spread becomes larger than $\rho$, which therefore puts an upper limit on the laser induced modulation that can be imprinted on the beam. This in turn, sets the highest harmonic.

QHG CONCEPT

Here, we outline a recently proposed scheme referred to as quieted high gain harmonic generation (QHG) that enhances the power output from HGHG FELs and can also boost the accessible harmonic number [5]. This technique exploits collective longitudinal space charge (LSC) effects generated by the sharp periodic density peaks to re-linearize portions of the modulated e-beam peaks while preserving the harmonic bunching. The LSC effect takes place in a dedicated short drift or focusing section immediately downstream of the chicane.

The layout is shown in Fig. 1, as is the evolution of the phase space. The beam is first modulated in energy by the laser (Fig. 1a), and then propagates through a standard four-dipole chicane. This generates density bunching at the chicane exit (Fig. 1b), as well as LSC forces between density spikes. Electrons near each density peak receive an energy kick from the repulsive forces; those close in front of the peak experience a positive energy kick, while those close behind have their energy reduced. After a short drift, the result (Fig. 1c) is a beam with sharp density spikes that coherently seed the HGHG process, but with a reduced projected energy spread between the spikes that facilitates lasing up to full saturation power.

Scaling

The effect is described in a simplified model where the beam is much longer than the laser wavelength and the transverse motion of particles is negligible assuming that the physical drift length is less than $\gamma\langle\beta \rangle/hk\epsilon_n$, where $\langle\beta \rangle$ is the average beta function and $\epsilon_n$ is the normalized emittance. In this regime, the general equations that describe evolution of the electron energy $\eta = \Delta\gamma/\gamma$ and longitudinal position $s$
in the e-beam frame are given as [6],

\[
\frac{d\eta}{dz} = \frac{q}{\gamma mc^2} E_z \quad \frac{ds}{dz} = \frac{\eta}{\gamma^2}
\]  

(2)

where \(-q\) is the charge of an electron, \(n_0\) is the beam volume density and \(E_z\) is the longitudinal space charge field. In a uniformly filled pencil beam with radius \(r_b\), electrons inside the microbunched beam experience LSC fields given by

\[
E_z(s, R) = \frac{2qn_0}{\varepsilon_0 k} \sum_{n=1}^{\infty} \frac{b_n}{n} \sin(n k s) F_n(R, \xi)
\]  

(3)

where \(R = r/r_b \leq 1\), \(\xi = k r_b / \gamma\), and

\[
F_n(R, \xi) = 1 - n \xi I_n(n \xi R) K_1(n \xi)
\]  

(4)

quantifies the contribution from the finite radius [7]. In the high frequency limit \(\xi \gg 1\), the 1D model is retrieved as \(F_n(R, \xi) \rightarrow 1\). Combining expressions and rescaling variables as \(p = \eta / (\sigma \eta_0)\), \(\tau = k_p z\), \(k_p^2 = q^2 n_0 / m e \varepsilon_0 c^2\gamma^3\), and \(\theta = ks\), the coupled equations that describe the evolution are

\[
\frac{dp}{d\tau} = 2 \alpha \sum_{n=1}^{\infty} \frac{b_n}{n} \sin(n \theta) F_n(R, \xi),
\]

\[
\frac{d\theta}{d\tau} = \alpha p.
\]

We have also defined

\[
\alpha = \frac{k \sigma \eta_0}{k_p \gamma^2},
\]

(6)

which is the energy spread parameter and sets the scale for the overall dynamics. It can be interpreted as the ratio of the longitudinal displacement due to thermal motion in a plasma period to the laser wavelength. The QHG regime requires \(\alpha < 1\) and the thermal motion can be neglected on the time scale of the plasma period [8].

The QHG scaling is obtained by linearizing the dynamical equations for small changes in the phase space position of a particle during the drift. The harmonic bunching is optimized for \(B = 1/A \ll 1\), and analysis shows that the dynamics are dominated by the lowest order LSC harmonic, i.e., \(b_1 = -1/2\). Thus, the drift length over which the initial energy modulation induced by the laser \(\Delta p = A\) is approximately canceled by space charge effects is then

\[
\Delta \tau \approx \alpha A / F_1(0, \xi).
\]  

(7)

The corresponding change in phase is approximately \(\Delta \theta \approx a h \Delta \tau \approx (a h)^2 / F_1(0, \xi)\) assuming \(A \approx h\) to obtain significant harmonic bunching. The natural dispersion of particles with different energies suggests that \(\Delta \theta \leq 1/h\) in order to preserve bunching at \(h\). This constrains the parameter \(\alpha\) to,

\[
\alpha < \sqrt{F_1(0, \xi) / h^3}.
\]  

(8)

Figure 2: Top: Initial phase space of an \(A = 10\) modulation, where WP particles (red) are overlay the rest of the particles (blue). Middle: Phase space at the end of a \(\Delta \tau \approx \alpha A / F_1\) (2 m) drift. Bottom: Energy spread along the drift of the whole beam (blue) and only the particles in the WP (red).

Figure 3: Top: Projected current distribution before the drift (solid line) and after (dashed). Bottom: Harmonic bunching spectrum before (solid line) and after (dashed) drift.
Simulations

To reveal the dynamics and features of the QHG process, we simulated the evolution of a cylindrical, uniformly filled beam in the 3D regime, governed by Eqs. (5). Results are shown in Figures 2 and 3 assuming $\xi = 1/2$ and $\alpha = 0.0065$. These parameters correspond to a $E=1$ GeV beam, for example, with radius $r_b = 37 \mu m$, $I_0=2kA$ of peak current, $\sigma_y = 2 \times 10^{-4}$ uncorrelated energy spread, and modulated by a laser with wavelength $\lambda = 240$ nm. The laser modulation amplitude is $A = 10$ and dispersion is $B = 0.1$, corresponding to $R_{36}=19 \mu m$. The initial and final phase space distributions are shown in the upper and middle plots in Fig. 2, respectively. In the lower plot, we see that over a physical drift length of 2.1 meters the energy spread of the electrons outside the density spike is strongly reduced by the LSC forces (red particles). We refer to this as the working portion (WP) of the beam where the bulk of the HGHG signal amplification takes place in the FEL, as seeded by the density spikes. Even though the energy spread of the whole beam is slightly increased in the drift (blue line), the HGHG process overall will be significantly enhanced by the quieted distribution in the WP. We note that the constraint that the change in phase be less than $1/\hbar$ is violated in this case, as evidenced by the shape of the final particle distribution and the two spikes in the current profile in Fig. 3. The beam has become slightly over-bunched in the extended drift section, which leads to a bifurcation of the initial single density spike (solid line) into two sharp spikes (dashed line). Note, however, that the bifurcated phase space generates bunching at even higher harmonics in the beam. Shown in Fig. 3, the beam initially has significant bunching at the 10th harmonic (solid line), as expected from the $A = 10$ modulation. Afterward, the final double spiked current profile has reduced bunching at the $h = 10$ harmonic (dashed line), but exhibits increased bunching at several higher harmonics.

The power output of this beam in a simulated HGHG FEL is shown in Fig. 4. The FEL is taken to be resonant at the 30th harmonic of the 240 nm laser, and the beam distribution from Fig. 2 (middle) was used for the QHG case. The FEL is seen to reach saturation at nearly full power using $\rho = 10\sigma y_0 = 2 \times 10^{-3}$. For comparison, the power output of a standard HGHG beam with an $A = 30 = 1/B$ modulation (to optimize the bunching at the 30th harmonic) is also shown. The power output of the HGHG beam is significantly reduced due to the large residual energy spread in the WP that has not been quieted by LSC effects.

CONCLUSION

We have provided a basic overview and supporting simulations of a quieted high gain harmonic generation mode of operation for FELs. Results show that the phase-locked LSC forces in a density modulated beam act to partially re-linearize regions of strong energy chirp in the beam. This facilitates lasing to saturation and access to higher harmonics for HGHG FELs.

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