EFFECTS OF BEAM LOADING AND HIGHER-ORDER MODES IN RF CAVITIES FOR MUON IONIZATION COOLING

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Abstract

Envisioned muon ionization cooling channel is based on vacuum and/or gas-filled RF cavities of frequencies of 325 and 650 MHz. In particular, to meet the luminosity requirement for a muon collider, the muon beam intensity should be on the order of $10^{12}$ muons per bunch. In this high beam intensity, transient beam loading can significantly reduce the accelerating gradients and deteriorate the beam quality. We estimate this beam loading effect using an equivalent circuit model. For gas-filled cavity case, the beam loading is compared with plasma loading. We also investigate the excitation of higher-order modes and their effects on the performance of the cavity.

INTRODUCTION

One of the main challenges in building muon collider is to achieve 6-D ionization cooling. A cooling channel should reduce the phase space occupied by the beam by about 6 orders of magnitude from the initial volume at the exit of the front end [1]. Two key 6-D cooling channel designs are currently under detailed study: vacuum cooling channel (VCC) with tapered rectilinear lattices [2], and helical cooling channel (HCC) based on gas-filled RF cavities [3]. In both channels, separate cooling channels are used for $\mu^+$ and $\mu^-$. In the first stage of the 6-D cooling channels, the emittance of a train of muon bunches (21 for the present design study) is reduced until the muon beams can be injected into a bunch merging system. The single muon bunches (one for $\mu^+$ and one for $\mu^-$) are then sent into the second stage of the 6-D cooling channel (see Fig. 1).

Although the number of muon per bunch is very high for both stages before and after the merge (see Table 1), most ionization cooling studies have not considered intensity-dependent effects. We note that before the merge, multibunch effects (beam loading and higher-order mode (HOM) wakefields) are of considerable concern, while after the merge, space-charge effects could be significant.

Figure 1: Functional elements of a muon collider complex.

BEAM LOADING ESTIMATION

Due to the high bunch currents ($I_b = 400 \sim 800$ A from Table 1), there are some concerns about beam loading. For muon collider applications, the muon beams are formatted with only 21 bunches in a pulse and the bunch spacing, $T_b = 1/(325\text{MHz})$ is short compared to the cavity filling time. Therefore, the beam loading does not reach a steady state, and the RF voltage seen by the last bunch will be considerably different from what the first bunch sees.

Method 1

In the previous study [4], we used an equivalent circuit equation in the slowly varying approximation to estimate the beam loading from the fundamental accelerating mode:

$$\frac{d\bar{V}_c}{d\tau} + \bar{V}_c = \bar{V}_f - \frac{1}{2}Q_L \left[ \frac{R}{Q} \right] I_b,$$

where $\bar{V}_c$ is the cavity voltage phasor which is also the sum of the forward ($\bar{V}_f$) and reverse ($\bar{V}_r$) voltages. Here, $\tau = t/T_f$ is time measured in units of filling time $T_f = 2Q_L/\omega_0$, and $Q_L$ is the loaded quality factor of the cavity.

For the case of a point-like bunch with the bunch length much shorter than the bunch spacing, the beam current for a high $Q_L$ cavity can be given by the single tone signal in phasor notation as follows:

$$I_b(t) \approx \text{Re} \left[ I_b(t) e^{i\phi_s} \right] \times H(t),$$

with $I_b(t) = 2I_{DC} e^{-i\phi_s}$ and $H(t)$ is the Heaviside step function. Here, $\phi_s$ is a synchronous phase and $I_{DC} = Q_b/T_b$ is the average DC current of a bunched beam of charge $Q_b$.

Combining Eqs (1) and (2), we can easily obtain the variation of the accelerating voltage $|\bar{V}_c| \cos \phi$ along the beam pulse at $t > 0$. Here, $\phi$ is the phase angle between $\bar{V}_c$ and $I_b$, which is $\phi = \phi_s$ at $t = 0$.

Method 2

In an alternative way, the beam-induced voltage can also be calculated by applying appropriate initial conditions to the 2nd-order homogenous R-L-C circuit equation for $t < 0$:

$$\left\{ \frac{d^2}{dt^2} + \frac{\omega_0^2}{Q_L} \frac{d}{dt} + \omega_0^2 \right\} V_b = 0.$$  

The initial conditions at $t = 0^+$ are

$$V_b(0^+) = -\frac{Q_b}{C},$$

where $C$ is the cavity capacitance.
Table 1: Example parameters for 6D cooling channel to provide $2 \times 10^{12}$ muons per bunch for multi-TeV muon collider. The number of muons is for only single sign.

<table>
<thead>
<tr>
<th>Channel</th>
<th>325 MHz</th>
<th>650 MHz</th>
<th>Bunch merge</th>
<th>325 MHz</th>
<th>650 MHz</th>
<th>End of channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission</td>
<td>0.84</td>
<td>0.84</td>
<td>0.8</td>
<td>0.84</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>Total number of muons</td>
<td>$11.8 \times 10^{12}$</td>
<td>$9.9 \times 10^{12}$</td>
<td>$8.3 \times 10^{12}$</td>
<td>$6.6 \times 10^{12}$</td>
<td>$5.6 \times 10^{12}$</td>
<td>$4.7 \times 10^{12}$</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>21</td>
<td>21</td>
<td>21 $\rightarrow$ 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bunch length (cm)</td>
<td>7.7</td>
<td>3.2</td>
<td></td>
<td>7.7</td>
<td>3.2</td>
<td>3.2</td>
</tr>
</tbody>
</table>

$$\frac{dV_b}{dt} \bigg|_{t=0^+} = -\frac{dQ_b}{dt} = \frac{1}{C} \frac{V_b(0^+)}{R}, \quad (5)$$

where the cavity resistance $R$ and capacitance $C$ are related to the unloaded quality factor $Q_0$ by $Q_0 = \omega_0 RC$. For $Q_L \gg 1$, the beam-induced voltage is approximated as

$$V_b \approx -2kQ_b \cos(\omega_0 t), \quad (6)$$

where $k$ is the loss parameter, which can be expressed in several different ways: $k = \omega_0 R/(2Q_0) = \omega_0 R_{sh}/(4Q_0) = dV_{sh}/dt|_{R/Q} = 1/(2C)$. Note that the negative sign in Eq. (6) indicates that the beam tends to decrease the accelerating voltage.

Figures 2 and 3 show that the two methods of calculating the beam loading for the fundamental mode yield nearly the same results. For the 325 MHz RF cavity the stored energy is rather high, so the overall beam loading is about 10%.

In this proceedings, making use of the analytical wake potentials calculated in Ref. [5], we address the effects of the higher-order modes. The wake potential for a highly relativistic point charge travelling along the $z$-axis at radius $r_b$ is

$$W_z(s; r_b, r_t, \theta_t) = 2 \sum_{mn\rho} k_{mn\rho}(r_b, r_t, \theta_t) \cos(\frac{\omega_{mn\rho}s}{c}), \quad (7)$$

where $\omega_{mn\rho}$ is the angular frequency of the $(m,n)$-th mode at frequency $\rho$.

Figure 3: Net accelerating voltages seen by 21 muon beam bunches. Here, $f_0 = 650$ MHz, cavity length $l_c = 26.9$ mm, $[R/Q] = 34.4$ Ohms, and transit time factor $T = 0.977$. The total number of muons is $9.9 \times 10^{12}$ for this case.

In the past, several analyses have been made to address wakefields in the muon cooling channel (see, e.g., Ref. [6]). In this proceedings, making use of the analytical wake potentials calculated in Ref. [5], we address the effects of the higher-order modes. The wake potential for a highly relativistic point charge travelling along the $z$-axis at radius $r_b$ is

$$W_z(s; r_b, r_t, \theta_t) = 2 \sum_{mn\rho} k_{mn\rho}(r_b, r_t, \theta_t) \cos(\frac{\omega_{mn\rho}s}{c}), \quad (7)$$

To understand the detailed structure of the higher-order modes, it requires a sophisticated time-domain electromagnetic simulation such as ACE3P/T3P (see, e.g., Fig. 4). When the beam is sub-relativistic regime as in the muon cooling channel ($\beta_\beta \approx 0.88$), it is more challenging as the radiated wakefields might catch up the source particles.

Figure 4: Preliminary calculation of the impedance spectrum excited by a Gaussian bunch with 1 cm rms length and $\beta_\beta = 1$ in a 650 MHz cavity.
where \( s > 0 \) is the distance between the source charge and a trailing charge. Here, \( k_{mn} \) is the loss parameter for mode \( TM_{mn} \):

\[
k_{mn} = \left( 2 - \frac{\delta m}{1 + \delta m} \right) \frac{1}{\pi \varepsilon_0 \omega_0^2 J_m(j_m) J_m(j_m)^2} \times \frac{J_m(j_m r_t)}{R_w} \frac{J_m(j_m r_t)}{R_w} \cos(m \theta_t) \times 2 \left[ 1 - (-1)^m \cos(\omega_{mn} l_c/c) \right]. \tag{8}
\]

For the special case of \( (r_t, r_j, \theta_t) = (0, 0, 0) \) and the fundamental cavity mode \( TM_{010} \), it can be easily shown that \( k_{010} \) becomes loss parameter \( k \) of the fundamental mode in Eq. (6), i.e.,

\[
k_{010} = \frac{1}{4} \frac{(l_c^2)}{\varepsilon_0 \omega_0^2 l_c^2 J_0(j_0)^2} = \frac{\omega_0}{4} \frac{R}{Q}. \tag{9}
\]

with \( T = \sin(\omega_0 l_c/2c)/\omega_0 l_c/2c \) and \( \omega_0 = \omega_{010} = j_0 l_c/R_w \). Here, \( R_w(l_c) \) is radius (length) of the cavity.

To evaluate the effect of the HOM, we consider the axisymmetric case only \( (m = 0) \). Figure 5 shows the total beam-induced voltages seen by the last bunch for the cases of \( n = 1, 2, 3 \) and \( p = 0 \) (left), and \( n = 1, 2, 3 \) and \( p = 0.1 \) (right). The fundamental mode term adds coherently and dominates at the end for \( p = 1 \) case. The total beam-induced voltages seen by the last bunch. Here, muons per bunch is \( 10^{12} \) and \( l_c = 10 \) cm.

**PLASMA LOADING**

For the HCC, in addition to the beam loading, plasma loading should be taken into account as well. The recent experimental results demonstrate that the effects of the plasma loading are in good agreement with the theoretical prediction, and furthermore they can be minimized by adding small amount of electronegative gas [7]. Based on these experimental observations, a rough calculation of the expected plasma loading in an actual HCC was done [9]. It was shown that the electrons decay quickly due to the attachment process, however ions build up over time. As the energy dissipation by the ions is much smaller due to their heavier masses, overall, no significant plasma loading is observed (see Table 2).

**SPACE-CHARGE EFFECTS**

Space-charge effects can be important after the bunch merge system. For VCC, it has been shown that the space-charge is not affecting the transverse emittance, but if the longitudinal emittance approaches ~1.5 mm, space charge is opposing additional cooling and causes particle loss in longitudinal phase-space [2]. For HCC, the longitudinal dynamics is above transition, so the longitudinal space-charge may act as a focusing force. In addition, we are investigating possible space-charge neutralization process due to the beam-induced plasmas in the gas-filled cavity (see initial result in Fig. 6).

![Figure 5: Preliminary calculation of the HOM’s in 650 MHz pill-box cavity. The red curves indicate 10% of the applied RF voltages (20 MV/m maximum), and the blue curves indicate the total beam-induced voltages seen by the last bunch. Here, muons per bunch is \( 10^{12} \) and \( l_c = 10 \) cm.](image)

![Figure 6: Interaction between muon beam (red) and electrons (green) showing pinching by self-magnetic field in the beam tail. Here, we assume muon beam is in the space-charged dominated regime, and electron dynamics is governed by its mobility. WARP Particle-In-Cell (PIC) code is used.](image)

**REFERENCES**


