Abstract
Precise and fast beam size measurement and emittance reconstruction in the different subsystems and transfer lines of the Future Linear Colliders (ILC and CLIC) will be essential for beam tuning in order to achieve the required luminosity. In this paper we investigate the feasibility of using a multi-Optical Transition Radiation (m-OTR) system for fast transverse beam size measurement, emittance reconstruction and coupling correction in the Ring to Main Linac (RTML) of the FLCs. Diagnostic sections of the RTML have been matched to the optimum optical conditions for emittance reconstruction. The necessary requirements for the OTR monitors to be placed in the RTML are discussed.

INTRODUCTION
The control and the preservation of low emittance along the RTML of the FLCs will be essential to obtain the required luminosity. Therefore, the diagnostic sections dedicated to emittance measurement and control are of special relevance for the RTML design.

In this context, the ILC RTML has been designed with seven diagnostic sections including laser wire scanners and other beam profile monitors [1]. A schematic of the ILC RTML is shown in Fig. 1, which includes the following sections: Ring To Linac (RTL), Long Transfer Line (LTL), Turnaround (TURN), Spin Rotator (SPIN), first and second stages of the Bunch Compressor (BC1/BC2) and their respective dump lines. These structures are present in both electron and positron beamlines.

In this paper we investigate the feasibility of using a m-OTR system for fast transverse beam size and emittance measurements at key places of the RTML, such as at the end of the BC2, prior injection to the Main Linac. In principle the OTRs can coexist with laser wire scanners. While the OTRs are intended for relatively low beam power use, the laser wire scanners will be necessary for beam size measurements and emittance reconstruction during the multi-bunch train operation. When operating with high charge and multi-bunch train the OTRs would be in non-measurement mode, i.e. they would be retracted from the beam path. OTR monitors could be very suitable for the setup and tuning of the machine in single-bunch mode, and can be very useful during startup and commissioning phases of the RTML.

It is worth mentioning that in the Accelerator Test Facility ATF2 at KEK [2] a four monitors based m-OTR system has shown to be very reliable and robust for performing fast emittance measurements and contributing efficiently to the tuning of the beamline. We wonder if a similar system could be applied to the context of the RTML. For the sake of comparison, Table 1 compares some relevant ATF2 beam parameters with beam parameters at the beginning and at the end of the ILC RTML. A preliminary OTR target damage study in the context of the ILC RTML was presented in [7]. In this paper we continue that study and we discuss optics conditions for an optimum emittance reconstruction.

EMITTANCE RECONSTRUCTION
The reconstruction of the projected emittance (2D) and the intrinsic emittance (4D) implies the computation of the entire beam matrix envelope $\sigma$, at a certain location of a beam line, which can be done from measurements and linear transformations of the beam distribution [3, 4]. These emittances are obtained by numerically solving three separated systems of coupled equations. The optics conditions for the existence and unicity of solutions of the systems of equations involved in the process of emittance reconstruction have been studied in detail in [5, 6]. In this section these conditions are summarized. Let us denote the measured beam sizes in the two transverse planes and the coupling term values by $\hat{\phi}^{(i)}$, $\hat{\phi}^{(i)}$, $\hat{\phi}^{(i)}$ respectively at the measurement stations labelled with $i = 1, 2, \ldots, N$, being $N$ the number of stations; the Twiss parameters at the measurements stations as $\rho^{(i)}$ and $\phi^{(i)} = \phi^{(j)} - \phi^{(i)}$ the phase advance differences between the measurement stations. In the general case of $N$ measurement stations:

Projected Emittance (2D) The first condition for reconstructing projected emittance (2D) is that the betatron...
phase advance differences between the measurement stations should not be an integer multiple of $\pi$:

$$\phi_{ij}^{(i)} \neq n\pi, \forall (i, j)$$  \hspace{1cm} (1)

This is the only required condition in the case of four measurement stations. For more than four stations, a second condition is required to get a unique solution:

$$-\delta_{1}^{(i)} \Delta_{3}(jkl) + \phi_{ij}^{(j)} \Delta_{3}(ikl)$$

This is the only required condition in the case of three measurement stations. For even more than four measurement stations, a fourth condition is required:

$$-\delta_{1}^{(i)} \Delta_{3}(jkl) + \phi_{ij}^{(j)} \Delta_{3}(iklm) - \delta_{3}^{(k)} \Delta_{3}(ijlm) + \delta_{3}^{(m)} \Delta_{3}(ijkl) = 0, \forall (i, j, k, l, m)$$

with:

$$-8 \left[ \beta_{x}^{(i)} \beta_{y}^{(i)} \beta_{x}^{(j)} \beta_{y}^{(j)} \right]^{-1/2} \Delta_{3}(ijkl) =$$

$$\cos(\phi_{x}^{(i)} + \phi_{y}^{(i)}) \left[ \cos(\phi_{x}^{(j)} + \phi_{y}^{(j)}) - \cos(\phi_{x}^{(j)} - \phi_{y}^{(j)}) \right]$$

$$+ \cos(\phi_{x}^{(i)} - \phi_{y}^{(i)}) \left[ \cos(\phi_{x}^{(j)} + \phi_{y}^{(j)}) - \cos(\phi_{x}^{(j)} - \phi_{y}^{(j)}) \right]$$

$$+ \cos(\phi_{x}^{(i)} + \phi_{y}^{(i)}) \left[ \cos(\phi_{x}^{(j)} - \phi_{y}^{(j)}) - \cos(\phi_{x}^{(j)} + \phi_{y}^{(j)}) \right]$$

$$+ \cos(\phi_{x}^{(i)} - \phi_{y}^{(i)}) \left[ \cos(\phi_{x}^{(j)} - \phi_{y}^{(j)}) - \cos(\phi_{x}^{(j)} + \phi_{y}^{(j)}) \right]$$

$$\neq 0, \forall (i, j, k, l)$$

respectively. Indices $i, j, \ldots$ refer to the stations. They take values from 1 to $N$ and should be different from each other. As discussed for Eq. (1) and Eq. (2), these conditions should be satisfied for any combination of 4 or 5 stations. Besides, the same argument concerning the experimental uncertainties of $\delta_{3}^{(i)}$ applies for Eq. (4).

**Intrinsic emittance (4D)**

The diagonalization of the beam matrix $\sigma$ gives us the intrinsic beam emittances (4D) $\epsilon_1$ and $\epsilon_2$:

$$\epsilon_{1,2} = \frac{1}{2} \sqrt{-Tr(J\sigma)^2 \pm \sqrt{(Tr(J\sigma)^2)^2 - 16det(\sigma)}} = \frac{1}{2} \sqrt{-Tr(J\sigma)^2 \pm \sqrt{(Tr(J\sigma)^2)^2 + 4Tr(J\sigma)^4}}$$  \hspace{1cm} (5)

**OPTICS LAYOUT AND TARGET DAMAGE STUDY FOR THE ILC RTL**

**The LTL Section**

Let us consider first the diagnostic section at the beginning of the Long Transfer Line (LTL) of the ILC RTL.

The optics functions of this diagnostic section is shown in Fig. 2. This lattice consists of FODO cells. It has four skew quadrupoles for cross-plane coupling correction and four laser wire scanners for projected emittance measurement. Here we propose to add four OTRs as indicated in Fig. 2 following the location pattern given in the conditions summarized in the section before and in [5] (condition1), that permits to make a complete reconstruction of the 2D and 4D emittances. This could be made without any change in the standard optics. A key aspect of the m-OTR system design is the target material. It must be robust enough to survive the beam impact in single bunch operation. Preliminary thermal studies [7] showed that targets made of aluminised Kapton polyimide film or beryllium (Be) could be suitable for OTR targets exposed to the RTML beams. Here we have reviewed those studies. Fig. 3 depicts the collision stopping power for electrons in different materials calculated using the well known Bethe-Bloch formula. In the case of the first diagnostic section of Fig. 2, the beam sizes at the OTR positions are: $\sigma_x \approx 64$ $\mu$m ($\beta_x \approx 5$ $m$, $\gamma \epsilon_x = 8$ $\mu$m at 5 GeV beam energy), and $\sigma_y \approx 4$ $\mu$m ($\beta_y \approx 7$ $m$, $\gamma \epsilon_y = 20$ $nm$ at 5 GeV beam energy). With these parameters the results of the instantaneous temperature rise (calculated following the
same procedure as in [7]) in Kapton and Be are shown in Table 2. It is necessary to mention that Kapton is a material that does not melt but decomposes at 793 K. In these case, for both Kapton and Be the temperature rise is bellow the damage limits.

Table 2: Collision stopping power \( (dE/dz) \), instantaneous temperature rise \( (\Delta T_{\text{inst}}) \), the thermal fracture limit \( (\Delta T_{\text{fr}}) \) and the thermal melting limit \( (T_{\text{melt}}) \) of the target material for an OTR in the diagnostic section at the beginning of the ILC LTL.

<table>
<thead>
<tr>
<th>Material</th>
<th>( dE/dz ) [MeV/cm]</th>
<th>( \Delta T_{\text{inst}} ) [K]</th>
<th>( \Delta T_{\text{fr}} ) [K]</th>
<th>( T_{\text{melt}} ) [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kapton</td>
<td>3.297</td>
<td>424.36</td>
<td>9240</td>
<td>–</td>
</tr>
<tr>
<td>Be</td>
<td>3.704</td>
<td>207.2</td>
<td>222.28</td>
<td>1546</td>
</tr>
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</table>

The BC2 Section

Let us now consider the case of the diagnostic section at the end of the ILC RTML, concretely at the end of the BC2. This diagnostic section contains four laser wires scanners with \( \pi/4 \) phase advance between them. Here we propose to place four OTR monitors around 30 cm apart downstream of each laser wire scanner, as shown in Fig. 4 (A). But in this case the standard optics has to be modified in order to de-phase the two planes following the location pattern given in the conditions summarized in the section before and in [5], that permits to make a complete reconstruction of the 2D and 4D emittances. Fig. 4 (B) shows an example of rematched BC2 diagnostic section to fulfill phase advance conditions (condition 3) for an efficient 4D emittance reconstruction.

![Figure 4: Optical layout of the diagnostic section at the end of the BC2 before the matching (A) and after the matching (B) to fulfil 4D emittance reconstruction requirements. The horizontal and vertical phase advance between the four OTRs is written in green and blue, respectively.](image)

In the same way as before we can estimate the temperature rise in a OTR target made of Kapton and Be, considering for example the optics configuration of Fig. 4 (B) and the OTR2 position: \( \sigma_x \approx 49.5 \mu m (\beta_x \approx 9 \ m, \gamma \epsilon_x = 8 \mu m \text{ at } 15 \text{ GeV beam energy}), \) and \( \sigma_y \approx 2.5 \mu m (\beta_y \approx 9 \ m, \gamma \epsilon_y = 20 \text{ nm at } 15 \text{ GeV beam energy}). \) The results are summarized in Table 3. In this case, \( \Delta T_{\text{inst}} \) surpasses the decomposition limit for Kapton, and it is about two times higher than the fracture limit for Be. Therefore to avoid damage it would be necessary to increase the betatron functions in this section by approximately a factor 2.

Table 3: Collision stopping power \( (dE/dz) \), instantaneous temperature rise \( (\Delta T_{\text{inst}}) \), the thermal fracture limit \( (\Delta T_{\text{fr}}) \) and the thermal melting limit \( (T_{\text{melt}}) \) of the target material for an OTR in the diagnostic section at the end of the ILC BC2.

<table>
<thead>
<tr>
<th>Material</th>
<th>( dE/dz ) [MeV/cm]</th>
<th>( \Delta T_{\text{inst}} ) [K]</th>
<th>( \Delta T_{\text{fr}} ) [K]</th>
<th>( T_{\text{melt}} ) [K]</th>
</tr>
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<tr>
<td>Be</td>
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<td>442.29</td>
<td>222.28</td>
<td>1546</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENTS

We gratefully acknowledge Dr. A. Vivoli for providing us with the latest ILC RTML optics (version 2012) in MAD8.

REFERENCES