SIMULATION OF LOW ENERGY CHARGED PARTICLE BEAMS*
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Abstract
Low energy particle beams pose specific challenges to simulation codes and experiments alike as a number of effects become important that can often be neglected at higher beam energies, including e.g. space-charge or fringe field effects. The optimization of low energy charged particle beam transport through arbitrary electromagnetic fields is the purpose of a code aimed at tracking low-energy particles from the sub-eV to the MeV energy range with high precision. The code is based on Matlab/Simulink and able to use 3-dimensional field maps from either Finite Elements Method (FEM) solvers, such as Comsol, OPERA 3D or CST particle studio, fields calculated by the code itself, or field maps from measurements. This paper describes the code structure and presents its performance limitations. It also gives a summary of results obtained from beam dynamics simulations of cyclotrons injection systems, storage ring extraction systems, electrostatic and magnetic beamlines, as well as from photocathode optimization studies.

INTRODUCTION
Accurate and efficient tracking codes are an important tool for designing particle accelerators as well as many other applications which use electromagnetic fields to control particles. Commercially developed tracking codes are often expensive, may appear to the user to be a black box and their performance is limited. An ideal code should be flexible and allow the initial particle position and velocity distributions to be defined by the user, as well as the particle type and the physics of beam interaction. On this basis a new code has been developed and has been applied to simulate different types of beams from low energy electrons and antiprotons used in fundamental physics research to ions used for medical treatment. It also allows performing beam tracking through a combination of electromagnetic fields, obtained either from simulations or measurements.

Maxwell’s equations are the backbone of all electromagnetic problems:
\[ \nabla D = \rho; \quad \nabla \times B = 0; \quad \nabla \times E = -\frac{\partial D}{\partial t} \quad \nabla \times H = j + \frac{\partial D}{\partial t} \]

General particle motion through electromagnetic fields can then be described by the equation:
\[ \frac{d\vec{V}}{dt} = \frac{q}{m} \left( 1 - \frac{V^2}{c^2} \right) \left( \vec{E} + \vec{v} \times \vec{B} \right) - \frac{1}{c^2} \vec{v} \times \vec{E} \quad (1) \]

CODE OVERVIEW
Like most other tracking programs, the here-presented code is based on the principle of step-by-step integration. Usually, time is used as the integration variable. With a set of given initial parameters and known electromagnetic fields one can carry out the numerical integration of equation (1), see Fig. 1.

The code has been written in Matlab-Simulink, allowing the built-in integration methods to be used. Commonly used methods are either Runge-Kutta (ode4) as a fixed-step method or Dormand-Prince (ode45) as a variable-step method, see Fig. 2.

One of the advantages of the code is its ability to set up any initial coordinates and velocities such as a Gaussian beam with specific Twiss parameters, a beam with particles distributed along the phase space ellipse for the purpose of beam line tracking or a beam with an initial energy distribution following Boltzmann’s law e.g. for electron emission studies.

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Specific physics problems require different approaches when it comes to the definition of the fields determining the motion of charged particles. The here-presented code allows integrating field maps from different sources, as well as calculating the fields during the tracking process.

![Diagram](image.png)

Figure 3: Illustration of the ways how field information can be integrated into the code.

Currently the code allows integration of field maps from measurements, analytical calculations (only possible for simple electrode or magnet geometries) and numerical maps obtained from FEM solvers.

![Diagram](image.png)

Figure 4: Comsol Multiphysics model of an electrostatic quadrupole.

There are a number of commercial and free software packages that are based on the finite element method (FEM) to solve electro-magnetic problems. The field maps calculated by these software packages can easily be imported into Matlab and then be used in the developed code. FEM solvers provide the electro-magnetic field values on the grid nodes which, depending on the grid type, can vary, see Fig. 4. There are two ways to export the field maps, either as a structured array which requires interpolation in specific locations or as values on the grid nodes, i.e. a scattered data array.

The former allows manipulation of the field map in the code. However, the pre-interpolation usually causes a negative effect on the achievable accuracy. The latter method is preferable as it provides raw data directly from the FEM solver. Both, the interpolation and extrapolation methods of 2D and 3D scattered data are included in the here-presented code.

Beam tracking through beamlines requires tracking of the beam in small steps through the full phase space. Besides a static coordinate system a second one is used that follows the beam through any turns in any direction. It allows calculation of the Twiss parameters of the beam at any step during the simulation.

**APPLICATIONS**

The code has been used successfully for simulations of various elements of accelerators, including medical, industrial and research cyclotrons, injection/extraction systems of low-energy electrostatic rings, beam transport lines, as well as experimental setups for electron emission studies. The following paragraphs outline the code’s application potential through several case studies.

The compact superconducting isochronous cyclotron C400 will be the first cyclotron in the world capable of delivering protons, carbon and helium ions for cancer treatment. A model of the Dee tip geometry at the cyclotron center with the inflector placed inside the housing was developed, see Fig. 5.

![Image](image.png)

Figure 5: Central region of the C400 cyclotron with its spiral inflector.

Beam dynamics simulations were carried out for beams with different initial distributions in transverse phase space planes obtained from the axial injection line. In these simulations a combination of electric fields from Comsol Multiphysics and magnetic fields from OPERA were used.

Beam extraction from a trap is a difficult task due to the strong magnetic fringe fields. Ways of optimizing the extraction from the MUSASHI trap at the Antiproton Decelerator (AD) at CERN were studied. In particular opportunities for further use of the beam in a small recycler ring [1]. In this simulation the magnetic field from an OPERA model of the trap’s solenoid field was used and combined with the 2D fields of the extraction electrodes that were designed by the group, see Fig. 6.

![Image](image.png)

Figure 6: Beam tracking through the electrostatic extraction system of the Musashi trap.
The Clatterbridge Cancer Center (CCC) in the UK provides eye melanoma treatment with a 60 MeV proton beam. The beam emittance at the beginning of the transport line from the cyclotron to the treatment room was measured in experimental studies [2]. Using the measured beam emittance and the obtained Twiss parameters tracking studies through the whole beam line were carried out, including 3 magnetic quadrupole triplets and a magnetic dipole, see Fig. 7.

Figure 7: Beam tracking through the CCC treatment beam line.

Here, the analytically calculated fields of the magnetic quadrupoles and dipole were used.

The code was also used to simulate the electrons that are emitted from a photocathode passing through several grid electrodes. By combining 2D and 3D field maps, calculated in Comsol Multiphysics a full field map of the experimental setup was successfully simulated [3], Fig. 8.

Figure 8: Electron emission from a photocathode.

Finally, for the electrostatic beamlines of the ELENA storage ring at CERN 3D electrostatic field maps calculated in Comsol Multiphysics were used together with analytical solutions [4] to increase the accuracy of the model. This was used as a basis for beam tracking with the goal to compute highly accurate transport matrixes of all ion optical elements, such as spherical deflectors, a fast kicker and different combinations of electrostatic quadrupoles. This kind of study requires high precision of the beam tracking, see Fig. 9.

Figure 9: Beam tracking through a series of electrostatic elements.

CONCLUSION

A new beam tracking code has been developed combining Comsol Multiphysics with purpose-written Matlab routines. This simulation environment is able to provide very accurate tracking through different electromagnetic fields. Its flexibility allows to solve different types of problems for different particles species from electrons to heavy ions. For low-energy particles our code provides very high accuracy and is efficient in terms of simulation times. Although it is aimed to deliver high accuracy for low-energy particle beams, it has shown to perform well in the MeV energy range. In this paper the structure of the code and its advantages were shown and examples of simulations already performed were given.

REFERENCES