SINGLE PARTICLE TRACKING FOR SIMULTANEOUS LONG AND SHORT ELECTRON BUNCHES IN THE BESSY II STORAGE RING

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Abstract
A scheme where 1.5 ps and 15 ps long bunches (rms) can be stored simultaneously in the BESSY II storage ring has recently been proposed (BESSY\textsuperscript{VSR} [1]). This paper presents simulations of single particle beam dynamics influenced by superconducting cavities used for the strong longitudinal beam focusing. The effect of RF jitter on (very short) bunches is investigated and results are discussed. Furthermore, possible effects on beam dynamics during ramp up and ramp down of the high gradient fields in the cavities are studied. The primary goal is to reveal preliminary design specifications for RF jitter on the basis of single particle dynamics.

INTRODUCTION
Since over 10 years, BESSY II successfully operates dedicated shifts with short bunches of 3 ps length, achieved by an optics with reduced momentum compaction factor \(\alpha\) [2]. The current in this so called low-\(\alpha\) optics is typically reduced to 40 \(\mu\)A per bunch to avoid the bursting instability induced by the coherent synchrotron radiation [3]. The instability does not cause beam loss but leads to an increase in bunch length and energy spread. As this operation mode conflicts with the interest of the majority of users, who rely on high photon flux from synchrotron radiation, the low-\(\alpha\) shifts are limited to 12 days per year. Half of those shifts are offered with 270 \(\mu\)A per bunch at an acceptable bursting level.

BESSY\textsuperscript{VSR} (A Variable Pulse Length Storage Ring) is an upgrade proposal for BESSY II presented in [1] with focus on short pulses and high flexibility and availability of various pulse lengths and fill patterns. This scheme overcomes the limitation of dedicated low-\(\alpha\) shifts, providing short and long bunches simultaneously at all beam ports. The transverse beam dimensions are identical to those presently applied in BESSY II.

In this scheme, a set of two sc cavity systems with a total RF gradient 100 times greater than the present setup are placed in a straight section of BESSY II. The cavities are operated at the 3rd harmonic (1.5 GHz) and 3.5th harmonic (1.75 GHz) of the fundamental RF cavity, as depicted in Fig. 1. In this paper, all cavities are assumed to be located at the same position in the ring, where the dispersion vanishes. The phase and accelerating voltage (25 MV and 21.4 MV) of the sc cavity systems is chosen in such a way that their gradients \(dV/dt\) add up at even bucket locations (e.g. \(t = 0\)) and cancel at odd bucket locations (e.g. \(t = 2\) ns), thus forming RF buckets for the short and long bunches respectively. Close to the position at \(t = 2\) ns, the RF potential is determined by the normal conducting cavity system, providing a zero current bunch length of \(\sigma_t = 10\) ps (rms), while at even bucket locations \(\sigma_t = 1\) ps is obtained. In BESSY II, a current dependent bunch lengthening of 50% just below the bursting threshold compared to the zero current bunch length has been observed. Other settings are also possible, e.g. a case where all cavity voltages cancel at \(t = 2\) ns, thus providing a highly non linear RF potential.

![Figure 1: Accelerating voltage versus time. Voltages of the fundamental RF cavity (green), the 1.5 GHz cavity system (red), the 1.75 GHz cavity system (blue) and the sum (black) are drawn. The ellipses indicate the locations of the short bunch (\(t = 0\) and \(t = 4\) ns) and the long bunch (\(t = 2\) ns).](image)

From single particle beam dynamics, the scaling of the bunch length is known to be \(\sigma_t \propto \sqrt{\alpha/(dV/dt)}\). Thus, the bunch length is decreased by a factor of 10 if the gradient is increased by a factor of 100. As presented in [1], the threshold current of the bursting instability is proportional to the RF gradient, allowing to store much higher currents in the scheme of BESSY\textsuperscript{VSR} than presently at the same bunch length. For a bunch length of 1.5 ps this results in a single bunch current of 800 \(\mu\)A. If the low-\(\alpha\) optics is combined with the proposed RF upgrade, BESSY\textsuperscript{VSR} will allow for bunches with 300 fs at 20 \(\mu\)A per bunch. The 200 long and 200 short buckets open up the possibility of various fill patterns and distributions of current in the long and short bunches up to a total current of 300 mA, limited by radiation safety.

RF JITTER
The suggested operation modes in BESSY\textsuperscript{VSR} imply very short bunch lengths, thus making the setup presumably sensitive to RF jitter, which will be discussed in this

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section.

All simulations were performed with the tracking code elegant [4], using canonical kick elements. Long term tracking (60–200 damping times of $\tau_s \approx 8$ ms, $1/\tau_s \approx 125$ Hz) with synchrotron radiation was performed to obtain longitudinal parameters in the equilibrium state. In all simulations, the jitter was applied on the 1.75 GHz cavity only. The cavity voltage $V(t)$ was modulated as $V(t) = V_0 \sin(2\pi f t) + A_\phi \sin(2\pi f t)$ to simulate phase jitter and $V(t) = V_0 (1 + A_E \sin(2\pi f t)) \sin(2\pi f t)$ to simulate amplitude jitter at a given jitter frequency $f$ with $V_0$ being the nominal voltage, $f_{RF}$ the RF frequency of the cavity, $A_\phi$ the amplitude of the phase jitter and $A_E$ the amplitude of the amplitude jitter.

The base values of the simulation are $A_\phi = 0.02$ deg and $A_E = 1 \cdot 10^{-4}$ respectively, which were found to be conservative upper limits for a single frequency jitter. The choice was made by evaluating jitter measurements of a TESLA cavity [5], shown in Fig. 2. The data was taken by a ring sampler of the LLRF system with high feedback gain.

First of all, the simulations showed that an amplitude jitter with an enhanced amplitude up to $A_E = 1 \cdot 10^{-2}$ has no measurable effect on bunch length or energy spread, thus, the following discussion focuses on phase jitter.

The bunch lengthening caused by a single frequency jitter shows no dependence on the jitter frequency $f$ for values well below the synchrotron frequency $f_s$ of the particular bunch. This relation is depicted in Fig. 3 left panel for the long bunch with $f_s \approx 8$ kHz. For $f$ well below $f_s$, the energy spread remains constant, even being independent of the jitter amplitude, as shown in Fig. 3 right panel. However, at frequencies around $f_s$, the synchrotron oscillation is resonantly driven which leads to a significant increase in the bunch lengthening and the energy spread.

The phase jitter was investigated at $f = 700$ Hz with the amplitude equal to the base value and by taking the measured data for $A_\phi$ (Fig. 2 top left panel) as a full spectrum jitter. In both cases, the spectrum was enhanced by a factor of 10 and 100 to estimate limits.

### Results

The results of the jitter studies are shown in Table 1. Generally, it can be said that for realistic amplitudes neither a single frequency phase jitter nor a full spectrum phase jitter affects the bunches negatively. In all cases (bunch lengths 300 fs, 1 ps, and 10 ps), no bunch lengthening can be observed within the statistical limits of the simulation, with the exception of the full spectrum jitter for the long bunch. There, a bunch lengthening and an increase in energy spread can be observed because the spectrum has a non-negligible line at $f \approx f_s$, Fig. 4 depicts this behavior in comparison with a single frequency jitter.

**Table 1: Simulation results of phase jitter with amplitude $A_\phi$ at $f = 700$ Hz and with a full spectrum.**

<table>
<thead>
<tr>
<th>$f_s$/kHz</th>
<th>bunch</th>
<th>$A_\phi$/deg</th>
<th>$A_\phi$/ps</th>
<th>$A_\phi$/10</th>
<th>$A_\phi$/100</th>
</tr>
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<tr>
<td>$\approx 17$</td>
<td>300 fs</td>
<td>0.33</td>
<td>8.5</td>
<td>1.0</td>
<td>7.1</td>
</tr>
<tr>
<td>81.0</td>
<td>1 ps</td>
<td>0.35</td>
<td>8.3</td>
<td>1.0</td>
<td>7.2</td>
</tr>
<tr>
<td>8.1</td>
<td>10 ps</td>
<td>0.37</td>
<td>8.1</td>
<td>1.1</td>
<td>7.3</td>
</tr>
<tr>
<td>$\times 10$</td>
<td>no jitter</td>
<td>1.2</td>
<td>8.7</td>
<td>1.5</td>
<td>7.1</td>
</tr>
<tr>
<td>$\times 100$</td>
<td>$A_\phi$ spectrum</td>
<td>0.34</td>
<td>8.0</td>
<td>1.0</td>
<td>7.0</td>
</tr>
<tr>
<td>$\times 100$</td>
<td>$A_\phi$ spectrum</td>
<td>1.0</td>
<td>19</td>
<td>1.0</td>
<td>7.0</td>
</tr>
<tr>
<td>$\times 100$</td>
<td>unstable</td>
<td>2.5</td>
<td>15</td>
<td>96</td>
<td>146</td>
</tr>
</tbody>
</table>

**RF RAMPING**

Due to the long bunches provided by the booster, the beam accumulation at full energy is expected to be performed with the sc cavities turned off. This investigation focuses on the increase of energy spread when the cavities are ramped up. The latter occurs because the RF bucket changes and the bunch is momentarily not in the equilibrium state. The energy spread has to be kept in limits to avoid loss of halo particles which already populate regions.

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**Figure 2:** Measurements of field stability of a TESLA cavity [5]. Left panels: Amplitude of phase jitter (top) and voltage jitter (bottom). Right panels: Integrated spectra yielding rms jitter values.

**Figure 3:** Bunch length (left panel) and energy spread (right panel) as a function of the jitter frequency at two different amplitudes of phase jitter.

**Figure 4:** Fourier transform of particle revolution time $t$ for long (blue line) and short bunches (red line) for a phase jitter of $A_\phi = 0.2$ deg at $f = 700$ Hz (left panel) and for the full spectrum multiplied by a factor of 10 (right panel).
of high energy deviation. Fast ramping may also be desirable considering the fact that the synchrotron frequency changes with the voltage gradient, thus crossing potentially harmful resonances.

For each ramp, a bunch with 1200 non-interacting particles was tracked for 30 ms ($\approx 37500$ turns) and the bunch length and energy spread were computed at every turn. For simplicity, linear ramps with different time characteristics were chosen. Non linear ramps with adiabatic behavior are still to be investigated. The results are shown in Fig. 5 in the top and center panel along with the time evolution of the ramps in the bottom panel.

When ramping up, the bunch length decreases as expected from 10 ps to 1 ps while the energy spread $\sigma_\delta$ rises and then goes back to its equilibrium value at $\sigma_\delta = 0.7 \cdot 10^{-4}$. The time behavior at which both quantities approach their equilibrium states is determined by the longitudinal damping time of $\tau_s \approx 8$ ms. Assuming an increase of $\sigma_\delta$ by a factor two to be an acceptable limit, ramping could be done in 10 ms.

$$Z_{th}^l(f) = \frac{2EQ_s}{eNF\alpha\tau_s},$$  \hspace{1cm} (1)

Figure 5: Bunch length (top), energy spread (center) and accelerating voltage (bottom) versus time for linear ramping of both sc cavity systems in 1 ms (black), 2 ms (red), 5 ms (green) and 10 ms (blue).

**EXPECTED HIGH CURRENT EFFECTS**

In addition to single bunch instabilities, there are other aspects that may impose a limit on the current stored in BERSSY$^{VSR}$. The Touschek effect may lead to low lifetimes for settings with very short bunches. Additionally, a limit may arise from coupled bunch instabilities (CBIs), either longitudinal or transverse, driven by the interaction of the beam with higher order modes (HOMs) of the sc cavities.

The threshold impedance for longitudinal CBIs can, for example, be estimated by [6]

$$Z_{th}^l(f) = \frac{2EQ_s}{eNF\alpha\tau_s},$$  \hspace{1cm} (1)

with $E$ the beam energy, $Q_s$ the longitudinal tune, $e$ the elementary charge, $N$ the number of cavities, $f$ the frequency, $I$ the average beam current, $\alpha$ the momentum compaction factor, and $\tau_s$ the longitudinal damping time.

Figure 6 illustrates the challenges of high impedance HOMs with respect to longitudinal CBIs in comparison to a HOM damped 1.3 GHz 7-cell cavity design [7] recently under development for BERLinPro [8]. Around 1.3 GHz, the TM010 passband modes, which are necessarily of high impedance, dominate the spectrum of unwanted HOMs. The design depicted here may serve only for an estimation as the parameters (frequency, number of cells) do not match the requirements of BERSSY$^{VSR}$. The design of heavily HOM damped multi cell sc cavities and the issue of CBIs are topics currently addressed with particular focus, also with active exchange of knowledge with colleagues from BERLinPro.

Figure 6: Threshold impedance (lines) for different setups in comparison with the calculated HOM spectrum of a 1.3 GHz sc cavity design [7].

**CONCLUSIONS**

It has been shown that present setups of sc cavities and LLRF systems provide field stability sufficient for all operation modes presently considered in BERSSY$^{VSR}$. In most cases, the critical limit for jitter is well above realistic values. From the beam dynamics’ point of view, a quick voltage ramp for the sc cavities, as short as 10 ms, also appears to be feasible. Challenges concerning high current operations are recognized and are under investigation.

**REFERENCES**