THE IMPLEMENTATION OF EQUIPARTITIONING IN THE PROTON LINAC CODE PADSC
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Abstract

The high intensity accelerator projects place extremely stringent requirements on particle loss, since even very small losses can lead to unacceptably high levels of radioactivity that can hinder or prevent hands-on maintenance. Such losses are known to be associated with emittance growth and beam halo. Non-equipartitioning contributes a lot for emittance growth and beam halo. The present equipartitioning realization has assumed that the emittance and space charge force are keeping constant, which will induce errors. The implementation in the proton linac code PADSC does equipartitioning optimization according to the real space charge force and emittance in the quasi period lattice.

INTRODUCTION

The free kinetic energy can be expressed with the concept of temperature, and when the temperatures in longitudinal and transverse directions are equal the beam reaches equipartitioning state. For non-equipartitioned beam, the space charge force will lead to emittance or energy exchange even when it is rms matched [1]. This process will induce emittance growth or beam halo and should be avoided especially for high intensity accelerators. Nowadays, the equipartitioning optimization is based on the transportation of matched KV beam in uniform focusing channel. It assumes that the emittance and space charge force are keeping constant through the entire linac. For space charge dominated beam which fulfill the above conditions, the equipartitioning condition can be expressed with the relationship between emittance and phase advance without space charge force in longitudinal and transverse directions. The proton linac code PADSC is programmed to do dynamic design, multiparticle tracking and structure optimization. One of its main features is equipartitioning optimization which is done according to the real emittance and space charge force in the real quasi period lattice.

CODE IMPLEMENTATION

The equipartitioning condition can be expresses as $T=1$, where the energy anisotropy $T$ is defined as:

$$T = \frac{\varepsilon_{n,z} k_z}{\varepsilon_{n,x} k_x}$$  \hspace{1cm} (1)

Where $x$ and $z$ means the transverse and longitudinal direction respectively, $k_x$ and $k_z$ means the focusing strength with space charge force.

Traditional Equipartitioning Realization Manner

The Eq.1 can be simplified to Eq.2 if the beam satisfies the following conditions [2]:

1) The beam is space charge dominated;
2) The accelerating structure is uniform where the space charge force and normalized emittance are keeping constant;
3) the values for the bunch aspect ratio and for the ratio $b/a$ of drift-tube radius $b$ to bunch radius $a$ are in the range where the approximation $g = 2y_0 z_m / 3a$ for the geometry factor is valid;
4) The external force and space charge force are linear.

$$\frac{k_{z0}}{k_{x0}} = \sqrt{\frac{3\varepsilon_{n,z} - 1}{2\varepsilon_{n,x}}}$$  \hspace{1cm} (2)

Where $k_{x0}$ and $k_{z0}$ means phase advance per meter without space charge force, and they are related to the lattice structure only, $\varepsilon_{x0}$ and $\varepsilon_{z0}$ are normalized emittance in transverse and longitudinal directions which are assumed to be constant.

The traditional equipartitioning process is based on Eq.2 and the above assumptions. But the real lattice is quasi period and the beam is not always space charge dominated. So, the assumptions of Eq.2 are not always satisfied.

The new Equipartitioning Implementation in PADSC Code

The new equipartitioning implementation in PADSC code is based on Eq. 1. It adjusts the quadrupole strengths to make the phase advance with space charge force satisfy equipartitioning condition, during which process the normalized emittance grows as a result of the nonlinear accelerating field.

Matching is the basis of the equipartitioning concept. The main issues of the implementation are both “equipartitioning” and “matching”. Take the strength of the quadrupoles in the current period as variables, the solution of the equipartitioning condition is not unique. PADSC finds the suitable solution according to nonlinear programming algorithm. To keep the lattice in quasi periodic and the beam is matched, the object function is set to be the minimization of the maximum mismatch factor in $x$, $y$, $z$ directions:

$$\min f = \text{Max}(M_x, M_y, M_z)$$  \hspace{1cm} (3)
Or we may set it to be the minimization of the sum of the quadrupole strength difference (or sum of the squares) with respect to the initial lattice:

\[ \min f = \sum_{i=1}^{n} \Delta G_i \]  

\( (4) \)

Where \( n \) is the number of cells in one period.

The object function is subject to be:

\[ \sigma_x = \sigma_{x,design} \]  

\( (5) \)

where \( x \) means in transverse direction, \( \sigma_x \) is phase advance in one period, \( \sigma_x = k_x L \) with \( L \) period length, and \( \sigma_{x,design} \) the designed phase advance which satisfies \( T=1 \) in Eq.1.

The initial lattice can be got according to Eq.2, or user can import lattice parameters from other codes. The PADSC repeats the “equipartitioning” and “matching” process in the first period until it finds the matched beam and the corresponding equipartitioning lattice. Then in the following periods it does “equipartitioning” only.

**OPTIMIZATION RESULTS**

Take the first DTL tank of CSNS as an example. The lattice is FFDD and “n” in Eq.4 is 4. The input energy of the lattice is 3.0258MeV and the RF frequency is 324MHz. The input total normalized emittances in longitudinal and transverse directions are 742.95deg×KeV and 0.16198cm×mrad

**Results with Traditional Equipartitioning Method**

Optimize the lattice with the traditional equipartitioning manner which is based on Eq.2. The curves of phase advance with space charge force are not smooth, as shown in Fig.1. The vertical and the horizontal values are not equal even when they have equivalent input emittance.

**Results with New Equipartitioning Method**

Optimize the lattice with the new equipartitioning method which is based on Eq.1. The objective function in the first period is set to be Eq.4, while in the following period is Eq.3.

Because of the nonlinear force in accelerating gap, the normalized emittance is not constant. The evolution of the normalized emittance is shown in Fig.3. Both the longitudinal and transverse emittance grow, but the growth rate of the former is larger compared to the latter.

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**Figure 1:** The phase advance after traditional “equipartition” optimization.

**Figure 2:** The envelope after traditional “equipartition” optimization.

**Figure 3:** The evolution of the normalized emittance.

Take the variation of the normalized emittance and the space charge force into account, the phase advance of the beam from the new equipartitioning manner in every single period is shown in Fig.4.
After the equipartitioning process for the whole tank, the quadrupole strength change. The relative change of the quadrupole strength is less than 10%, as shown in Fig.6. The value under the new method (green points) is a little larger than that under the traditional method (red points).

The curves of factor T from the traditional and new equipartitioning methods are shown in Fig.7. From the Figure, we can see that the initial lattice which is based on Eq.2 is non-equipartitioning, while the result of the new method in PADSC is equipartitioned (T=1).

**CONCLUSIONS**

The implementation of equipartitioning in PADSC is presented. Its implementation is based on the real normalized emittance and space charge force in quasi period lattice. One optimization example is done and the results from the new method are improved compared with the traditional realization manner which is based on several assumptions. Still some details will be discussed and studied later, such as the influence of the first half quadrupole at the entrance of the tank.

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**REFERENCES**
