INTERACTION OF MUON BEAM WITH PLASMA DEVELOPED DURING IONIZATION COOLING

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Frontiers in particle accelerators/colliders:

- **ILC**: $e^+e^-$ machine ($\sim 31$ km, $E \sim 1$ TeV, $L \sim 2 \times 10^{34}$ cm$^{-2}$ s$^{-1}$)
- **CLIC**: $e^+e^-$ machine ($\sim 51$ km, $E \sim 3$ TeV, $L \sim 2 \times 10^{34}$ cm$^{-2}$ s$^{-1}$)
- **Muon Collider**: $\mu^+\mu^-$ machine ($\sim 2$ km diameter, $E \sim 3$ TeV, $L \sim 4 \times 10^{34}$ cm$^{-2}$ s$^{-1}$)

- **Plasma Wakefield accelerator** (PWFA) --- SLAC, UCLA, ANL
  (Beam produced plasma and Laser produced plasma)
- **Dielectric Wakefield accelerator** (DWA) --- SLAC, ANL
- **THz Radiation Technology** --- SLAC, ANL

**Muon Collider has unique feature !**

$m_\mu \sim 207m_e \implies$ low synchrotron radiation, small energy spread at IP
$\mu^+ - \mu^-$ Collider

\[ L \propto \frac{P_b}{E_{c.m.}} \times \frac{N_\mu}{\sigma_x \sigma_y} \]

- 8 GeV SC Linac
- Main Injector to 60 GeV
- Hg Target
- 20 T Capture Solenoid
- Phase Rotation to 12 bunches
- Linear Transverse Cooling
- 6 D Cooling
- Merge 12 to One Bunch
- 6 D Cooling
- Transverse Cooling in 50 T
- Linac
- RLA(s)
- Pulsed Synchrotron(s)?
- Collider Ring 1.5 / 4 TeV
Ionization Cooling Principle

Balance between cooling and heating gives normalized emittance:

\[
\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2} \frac{dE_\mu}{ds} \frac{\varepsilon_N}{E_\mu} + \frac{\beta_\perp (0.014 \text{ GeV})^2}{2 \beta^3 E_\mu m_\mu X_0} \frac{\beta_\perp (0.014 \text{ GeV})^2}{2 \beta m_\mu X_0} \frac{dE_\mu}{ds}
\]

\(\varepsilon_{x,N,equil.} = \frac{\beta_\perp (0.014 \text{ GeV})^2}{2 \beta m_\mu X_0} \frac{dE_\mu}{ds}\)
Muon Ionization Cooling – A Quick Look

MICE Experiment, UK

Efforts at Fermilab
Motivation

Important questions during the process of ionization cooling:

- How does the electron concentration of an absorbing material (e.g. \(10^{29}\) m\(^{-3}\) for Li) interact with the incoming \(\mu\)-beam?
- How does the high density of ionization developed in the process of cooling affects the incoming beam?

A realistic model should take into account the complete process, the ionization of medium due to incoming beam and its interaction. However, modeling the ionization of a medium at very high pressure (~ 10 – 100 atm) is complex (fluid + PIC code).

To study the collective effects in beam interaction with dense ionized medium, it is justified to assume that plasma has been formed by several bunches passed through the absorber. This model can address the above questions.
Important effects need to study:

- The excitation of plasma wave and wakefields for $\mu^-$ and $\mu^+$ beam propagating inside plasma.
- Effects of various densities of plasma on incoming beam.
- Effects of external magnetic field.
2-D self-consistent EM PIC code (FDTD), developed by Plasma Theory and Simulation group, UC Berkeley.

- **2-dimensional spatial grid**
  - Cartesian (x,y) or cylindrically symmetric (r,z)
  - Non-uniform grids in both dimensions

- **Plasma and beam emission / interaction**
  - Boundary interactions (absorption, reflection)
  - Secondary emission from boundaries
  - Monte Carlo scattering between species; ionization
  - Time-dependent current injection
  - Tunneling ionization

- **Space charge physics**

- **Full EM field solver**
  - Electromagnetic Problems
  - Wakefields -- PWFA and LWFA

- **Supports MPI implementation for distributed computing**

- **Application:** Microwave devices, Plasma sources, Beam optics, Laser/beam plasma interaction, Accelerators.
Parameters of Study

Beam:
Shape: Gaussian
Particle: muons ($\mu^-, \mu^+$)
$N_b = \text{number of beam particles} = 1 \times 10^{12} \text{ per bunch}$
$r_b = \text{bunch radius} = 3 \text{ mm}$
$L_b = \text{bunch Length} = 40 \text{ mm}$
$P = \text{reference momentum} = 200 \text{ MeV/c}$
$m_\mu = \text{rest mass of muon} = 105.7 \text{ MeV/c}^2$
$\beta = 0.88$
$\gamma = 2.1$
$\tau_p = \text{pulse length} = 151 \text{ ps}$
$E_{tot} = 226 \text{ MeV}$
$Q_b = \text{total charge} = 160 \text{ nC}$
$n_b = \text{peak beam density} \sim 10^{18} \text{ m}^{-3}$
$\omega_{pb} = \text{plasma frequency of beam} \sim 10^9 \text{ rad/s} (\sqrt{\frac{ne^2}{\varepsilon_0 m}})$

Plasma: Li
Density varies in the simulations ($10^{16} – 10^{22} \text{ m}^{-3}$)
**Simulation Setup**

2D Cylindrical Symmetry

Grid Space ($N_r \times N_z$) = 80 x 480

Time Step-size ($\Delta t$) = 0.2 ps (stability condition)

Beam Absorber Volume Ionized ($Li^+, e^-$)

Axis: Reflective Boundary
XOOPIC Simulation Setup at Time $t = 0$

- Muon beam enters the cold lithium plasma.
- Plasma ions are modeled as a stationary uniform background.
- Plasma electrons are modeled with uniformly distributed particles with zero initial velocity.

\[ n_b \sim 10^{18} \text{ m}^{-3}, \quad n_{pe} \sim 10^{18} \text{ m}^{-3} \]
Plasma Wakefield Excitation by $\mu^-$ & $\mu^+$ Beam

Time snapshot, pulse head @ 80 mm

Muon Beam

Electron Plasma

High e\textsuperscript{-} density

Wakefield
Evolution of $\mu^-$ - $\mu^+$ beam in e^- Plasma

$\mu^-$

$\mu^+$

wavelength of plasma wave = 32 mm for $n_e = 10^{18}$ m$^{-3}$

Plasma wave is excited, however, not dangerous
3-D Wake E-field Structures for $\mu^-$

Snapshot @ $t = 210$ ps ($\sim 55.4$ mm)  
Beam-length = 40 mm

Bunch head generates strong negative wakefield -- acceleration & compression

Bunch Tail

14.9 mm
Total Energy of $\mu^-$ and $\mu^+$ Beam
Collective Effects for $\mu^{-}$ Beam in Plasma
Effects of Plasma Density for $n_b=10^{18}$

Plasma Wiped out

Strong Wakefield

Weak Wakefield

Unaffected

$n_{pe} = 10^{16}$

$n_{pe} = 10^{18}$

$n_{pe} = 10^{20}$

$n_{pe} = 10^{22}$
Evolution of $\mu^-$ beam in Under and Over Dense Plasma

**Beam Window**

$n_b = 10^{18}$
$n_{pe} = 10^{16}$

**Plasma Window**

$n_b = 10^{18}$
$n_{pe} = 10^{22}$

Under dense plasma Wiped out, over dense plasma unaffected
Beam and plasma of almost equal density have strong interaction
Effects of Magnetic field ($B_z$) -- Movie

For $\mu$ collider, cooling channel is placed in a strong B-field

$B_z = 0.5$ T

$n_b \sim 10^{18}$, $n_{pe} \sim 10^{18}$

$B_z = 1$ T

Magnetic field suppresses the strength of wakefield
Conclusions

- Particle-in-cell simulations of beam interaction with plasma reveal detailed wakefield structures which depend on beam and plasma densities, applied field strength, polarity of beam particle, etc.

- Plasma wakefield excitation important when peak density of beam is comparable with plasma density, consistent with the other plasma wakefield accelerator simulations --- polarization of medium and wakefield does not stop beam.

- Negatively charged beam experiences net acceleration. However, acceleration is weak for positively charged beam. These results are consistent with SLAC and Max Planck Institute wakefield accelerator simulation results. *Wakefield due to $\mu^+$ is weaker than $\mu^-$*.

- External magnetic field can suppress wakefield which in turn may prevent longitudinal emittance increase.

- Present simulations reveal that **collective effects are not important** for the present design parameters.
Thanks for your Attention