A METHOD TO OBTAIN THE FREQUENCY OF THE LONGITUDINAL DIPOLE OSCILLATION FOR MODELING AND CONTROL IN SYNCHROTRONS WITH SINGLE OR DOUBLE HARMONIC RF SYSTEMS

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Abstract

In a heavy-ion synchrotron the bunched particle beams can perform longitudinal oscillations of several modes. These oscillations are damped by Landau damping, but can become unstable if driven accordingly. Furthermore, Landau damping is accompanied by filamentation which increases the longitudinal beam emittance and thus reduces the beam quality. To stabilize the beam and to keep the emittance low, control measures are taken. However, any controller design requires knowledge about the oscillation frequency of the mode which is to be damped. For a single harmonic RF voltage and a very small bunch, the longitudinal equations of motion of the particle can be linearized, and all the particles oscillate with approximately the same synchrotron frequency which equals the frequency of the dipole oscillation. For a larger bunch, or for a double harmonic RF system introducing nonlinearities around the reference point, this is no longer valid. In this work, we present a method to obtain the frequency of the dipole oscillation for larger bunches with a single or double harmonic RF system and show how this can be used for a state space controller design.

SINGLE AND DOUBLE HARMONIC RF VOLTAGE

In this paper a single and a double harmonic RF system

$$V_{sh}(\tau) = \hat{V}_1 \sin(\omega_{RF}\tau) , \quad (1a)$$

$$V_{dh}(\tau) = \hat{V}_1 \sin(\omega_{RF}\tau) + \hat{V}_2 \sin(2\omega_{RF}\tau + \Delta\varphi_{dh}) \quad (1b)$$

are considered, where $\hat{V}_2$ and $\Delta\varphi_{dh}$ are chosen in such a way that a saddle point in τ_R [7] occurs, where τ denotes the arrival time with respect to the zero crossing of the single harmonic sinusoidal component of the RF voltage. For the sake of simplicity only the stationary case below transition energy with $\varphi_R = \omega_{RF}\tau_R = 0$ is considered. Fig. 1 shows the different RF voltages.

Furthermore only low beam intensities are considered and therefore space charge effects introducing a synchrotron frequency tune shift [6] are neglected.

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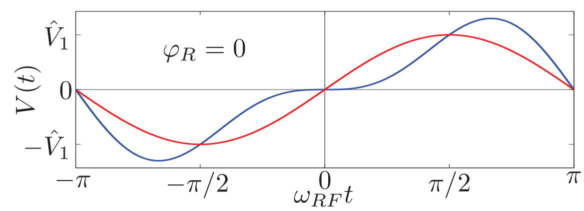

Figure 1: single and double harmonic gap voltage with identical first cavity voltage amplitude $\hat{V}_1$.

STATE OF THE ART

Using the phase space variables $\varphi = \omega_{RF}\tau$ and $\frac{\Delta E}{\omega_R}$ with $\Delta E = E - E_R$, the longitudinal equations of motion for a single particle can be linearized for $V(\varphi) = V_{sh}(\varphi)$ and short bunches [3]. In this case the barycenter of the bunch oscillates with the frequency

$$f_{syn,0} = f_R \sqrt{\frac{hq\hat{V}_1 |\eta_R \cos\varphi_R|}{2\pi\beta_R^2 E_R}} \quad (2)$$

with the harmonic number h, the phase slip factor $\eta_R = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma_R^2}$, the angular revolution frequency ω_R, the speed β normalized to the speed of light, the total energy E_R and the charge q of the particles.

In case of a realistic bunch length this linearization is no longer valid. As a result, the synchrotron frequency spread of the particles has to be taken into account and the rigid dipole mode does not oscillate with the linear synchrotron frequency, but with a smaller frequency ("*coherent synchrotron frequency*"). For small displacements $\varphi_B \ll \pi$ of the bunch barycenter a way to estimate the coherent synchrotron frequency is given e.g. in [1]:

$$\Omega_{syn,coh} = \omega_{syn,0} \sqrt{\frac{1}{|u_m|} \int_{\varphi_{m1}}^{\varphi_{m2}} \left(\frac{V(\varphi)}{\hat{V}_1}\right)^2 d\varphi}$$

with $u_m = \int_{\varphi_{m1}}^{\varphi_{m2}} (Y(\varphi) - Y(\varphi_{m2}))\, d\varphi$; $Y(\varphi) = \frac{1}{\hat{V}_1} \int_0^{\varphi} V(\varphi)\, d\varphi$; $\varphi_{m1,2}$: left/right end of the bunch. However, it is not specified how the end of the bunch is to be determined.

Another common way to obtain the synchrotron frequency for a larger bunch is the measurement of the beam transfer function [5]. However, these measurements are rather intricate and the obtained values are only valid for the very bunch size and ion type used in the experiment.

SINGLE HARMONIC OPERATION

Coherent Synchrotron Frequency for Large Bunches

In the following, the standard deviation of the bunch in direction of φ will be denoted by σ_φ. For different particle densities, the bunch length will be defined as $4\sigma_\varphi$. As the bucket length equals 2π, feasible values will be $\sigma_\varphi \in [0;\, \pi/2]$.

The nonlinear sinusoidal voltage leads to lower synchrotron frequencies of particles with a larger phase deviation φ which decreases the coherent synchrotron frequency. According to [3] the oscillation period of a particle with a maximum phase coordinate φ^* can be written as

$$f_{syn}(\varphi^*) = f_{syn,0}\,\frac{\pi}{2K\,(\sin(\varphi^*/2))}\,,$$

where $K(k)$ denotes the complete elliptic integral of the first kind. The oscillation frequency of the barycenter of the bunch can be calculated for different particle densities with a moment method that was developed in [4]. For a Gaussian density and the assumption that the bunch shape is approximately ellipsoidal, the frequency for small dipole amplitudes is derived as

$$f_{syn,coh,g}(\sigma_\varphi) = f_{syn,0}\,\sqrt{1+\sum_{n=1}^{7}\frac{(-1)^n}{n!\,2^n}\,\sigma_\varphi^{2n}}$$

and for an ellipsoidal bunch with a uniform density as

$$f_{syn,coh,u}(\sigma_\varphi) = f_{syn,0}\,\sqrt{1+\sum_{n=1}^{7}\frac{(-1)^n}{n!\,(n+1)!}\,\sigma_\varphi^{2n}}.$$

Fig. 2 compares the two coherent frequencies with the synchrotron frequency for the trajectory $\varphi^* = 2\sigma_\varphi$. It is apparent that as a rule of thumb, the synchrotron frequency at $2\sigma_\varphi$ is a good estimate for the coherent frequency of the barycenter, i. e.

$$f_{syn}(\varphi^* = 2\sigma_\varphi) \approx f_{syn,coh,u}(\sigma_\varphi) \approx f_{syn,coh,g}(\sigma_\varphi)\,.$$

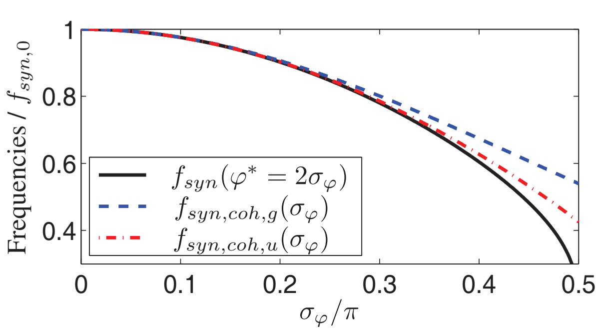

Figure 2: coherent frequency of the barycenter for a single-harmonic setup compared to the synchrotron frequency.

Fig. 3 shows the relative error of this estimate. For a bunch to bucket length ratio of up to $2\sigma_\varphi/\pi = 0.7$, the error of this rule is less than 5% for both Gaussian and uniform distributions. For a uniform distribution $2\sigma_\varphi$ equals the maximum phase deviation of the border of the bunch.

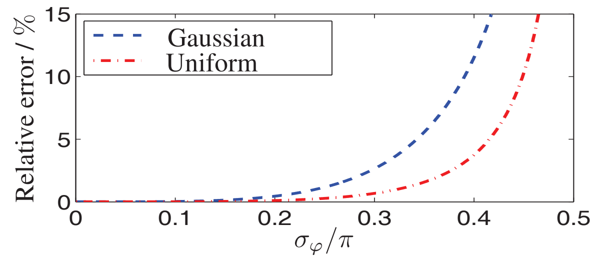

Figure 3: relative error if the coherent frequency is estimated with the synchrotron frequency for the trajectory $\varphi^* = 2\sigma_\varphi$.

DOUBLE HARMONIC OPERATION

For obvious reasons the equations of motion can not be linearized if a double harmonic voltage is applied. As a first approach to obtain the coherent synchrotron frequency also the $2\sigma_\varphi$-trajectory can be considered. The synchrotron frequency of a particle moving along a certain trajectory can be derived as

$$f_{syn}(\varphi^*) = f_{syn,0}\frac{\pi\sin\frac{\varphi^*}{2}}{\sqrt{2}K\left(\sqrt{\frac{1}{2}\left(1+\sin^2\frac{\varphi^*}{2}\right)}\right)} \tag{3}$$

as shown e.g. in [3]. Macro particle simulations show that the synchrotron frequency of the $2\sigma_\varphi$-trajectory is a good first estimate of the coherent synchrotron frequency but can still be improved. This can be seen in Fig. 5 which shows the percental error $\Delta f_{syn,coh} = \frac{f_{syn,coh,sim} - f_{syn}(2\sigma_\varphi)}{f_{syn,coh,sim}}$ of this approach along with an improved estimate which is presented below. In the simulation a matched bunch ($\sigma_\varphi = 0.28\pi$) was shifted by $\varphi_B(t=0) = 0.07\pi$.

To obtain a better estimate for the coherent synchrotron frequency at first macro particle simulations are performed for several bunch sizes σ_φ and barycenter deviations $\varphi_B(t=0)$. The simulated coherent synchrotron frequencies are compared to the synchrotron frequencies along the trajectories in phase space, and for each simulation the trajectory is determined whose synchrotron frequency equals the coherent synchrotron frequency. This yields a value of φ^*_{opt} for each bunch size σ_φ and phase deviation φ_B which, inserted in Eq. (3), results in the coherent synchrotron frequency. The simulated values of φ^*_{opt} for ^{40}Argon^{18+} ions with a kinetic energy of $E_{kin} = 11.4\frac{\text{Mev}}{\text{u}}$ are shown in Fig. 4.

By introducing a smoothing function, φ^*_{opt} can be expressed in terms of σ_φ:

$$\varphi^*_{opt} \approx -0.4287\frac{\sigma_\varphi^2}{\pi} + 2.1898\sigma_\varphi\,. \tag{4}$$

This improves the percental error of the estimate significantly, as can be seen in Fig. 5. Similar results are obtained for different ion types and beam energies.

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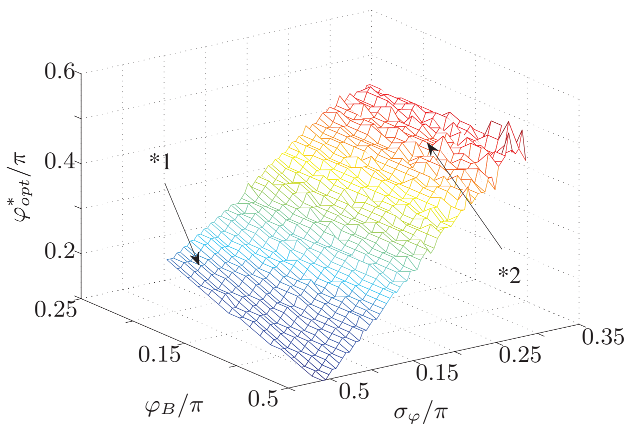

Figure 4: simulated values of φ^*_{opt} for different bunch sizes σ_φ and barycenter deviations φ_B. ***1:** For unrealistic small bunches with a large barycenter deviation, φ^* depends on φ_B. ***2:** For realistic bunch sizes φ^* depends only on σ_φ .

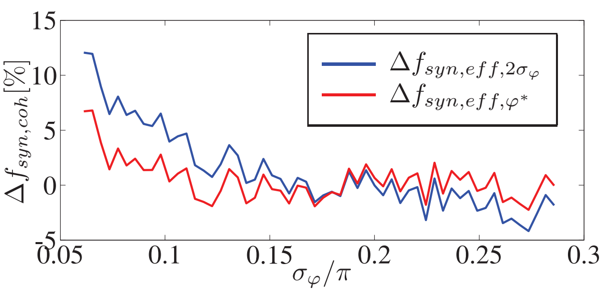

Figure 5: percental deviation of synchrotron frequency of the $\varphi^* = 2\sigma_\varphi$-trajectory and φ^*_{opt} according to smoothing function Eq. (4).

CONTROLLER DESIGN

To damp the dipole oscillation a state space controller is designed based on the model of a harmonic oscillator with the coherent synchrotron frequency. The two variables are the bunch barycenter phase φ_B and the energy deviation ΔE_B. The actuating variable is the phase shift φ_{gap} of the accelerating voltage e.g. for a double harmonic system

$$V_{dh}(\varphi) = \hat{V}_1 \sin(\varphi - \varphi_{gap}) + \hat{V}_2 \sin(2\varphi - 2\varphi_{gap} + \Delta\varphi_{dh}) . \quad (5)$$

In the SIS 18 at GSI the beam current is measured from which $\varphi_B - \varphi_{gap}$ is obtained [2]. The cavities are synchronized to direct digital synthesis (DDS) modules, which means that in order to calculate φ_B a second DDS has to be run to obtain the phase shift φ_{gap} of the RF voltage. As the energy deviation ΔE_B can not be measured, an observer is needed. The controller performance is improved if $\varphi_B - \varphi_{gap}$ is fed back which smoothens the progression of the bunch barycenter. An additional integrator is added as a countermeasure to a remaining offset. The closed control loop is shown in Fig. 6.

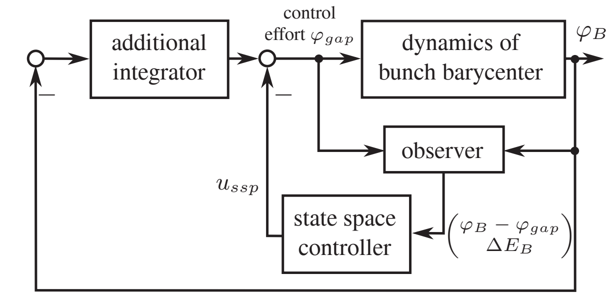

Figure 6: closed control loop: bunch barycenter $(\varphi_B,\ \Delta E_B)^T$, control effort of state space controller $u_{ssp} = -\underline{k}^T \cdot (\varphi_B - \varphi_{gap},\ \Delta E_B)^T$.

The controller is optimized for robustness against parameter uncertainties and keeps the beam emittance to a minimum, as shown in Fig. 7. The simulation parameters were $\sigma_\varphi = 0.28\pi$, $\varphi_R = 0$, double harmonic RF system with $\hat{V}_1 = 16000\text{V}$, ${}^{40}\text{Argon}^{18+}$ ions with a kinetic energy of $E_{kin} = 11.4\frac{\text{MeV}}{\text{u}}$, $\varphi_B(t = 0) = 0.1\pi$.

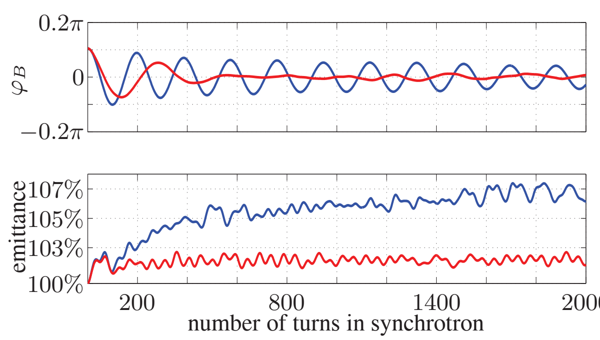

Figure 7: phase deviation of bunch barycenter and beam emittance growth in the open loop and in the closed loop case.

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[7] Throughout this paper the index $_R$ indicates the reference particle.