

HOW TO ACHIEVE LONGITUDINALLY POLARIZED ELECTRONS USING INTEGER SPIN TUNE RESONANCES*

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Abstract

Commonly, strong solenoids are used in circular accelerators to achieve longitudinal polarization. In practice, however, these solenoids cause a phase space coupling, which has to be compensated for by sophisticated decoupling schemes. We suggest to adiabatically ramp into an integer spin tune resonance while preserving the degree of polarization. When appropriately adjusting the driving horizontal field contributions at the final energy, the resulting polarization is longitudinal at predefined positions in the accelerator. Here, depending on the energy spread, the degree of polarization is conserved for several seconds. The contribution shows the numerical analysis of this scenario being confirmed by first demonstration tests at the ELSA stretcher ring.

INTRODUCTION

The invariant spin field $\hat{n}(\bar{u})$ depends on the 6-dimensional phase space \bar{u} and describes a spin configuration which stays invariant turn by turn. It was introduced by Derbenev and Kondratenko [1] in order to describe an equilibrium state of a spin ensemble inside circular accelerators. For $t \rightarrow \infty$, an ensemble of ultrarelativistic electrons aims to achieve the mentioned equilibrium state. Here, the emission of synchrotron light is the crucial factor which leads to spin diffusion towards the invariant spin configuration. Analyzing the invariant spin field is interesting not only for the asymptotic spin configuration, but also for investigations in smaller time scales. In the range of hundreds of revolutions, the polarization vector follows the invariant spin axis $\hat{n}(\bar{0})$ adiabatically, as long as it is initially aligned to the direction of the invariant spin axis. Hereby, the direction of the polarization vector can be adjusted in the horizontal plane as accurately as it is possible for the invariant spin axis. For this purpose, it is appropriate to use horizontal magnetic fields which excite an integer spin tune resonance. If one adiabatically increases the kinetic energy of the particles towards to the resonance condition, the polarization vector will follow the invariant spin axis from the originally vertical into the longitudinal direction. Besides, one has to pay attention to the resulting depolarization. Since the spin tune varies for different particle momenta, the precessions become dephased and the spins lose their coherent motion. Even though the asymptotic degree of polarization will be zero due to the spread of the kinetic energy of the electrons, the degree of polarization can largely be preserved for at least a few seconds.

First, we illustrate schematically how to obtain longitudinal polarization using a simplified method for the calculation of the invariant spin axis \hat{n}_0 on the closed orbit. These calculations are followed by a detailed numerical spin tracking which will show how long the polarization can be preserved for the third integer spin tune. In addition, we show first empirical demonstration tests for the stretcher ring of the ELSA facility at the mentioned resonance. The numerical studies and examples are based on typical ELSA settings, but can also principally be extended for other lattices.

ILLUSTRATION OF PROPOSED METHOD

Near to an isolated resonance, the invariant spin axis rotates slowly into the horizontal plane during the increase of the particle energy towards an integer spin tune. Hence, the polarization vector follows the axis adiabatically. This is true if one only considers the center of charge motion¹ and if no synchrotron motion is considered. The integer spin tune resonance is either driven by fields due to unavoidable misalignments and field errors or can be excited by enforced field distributions e.g. using horizontal dipole correctors. Therefore, one should be able to obtrude the direction of the invariant spin axis on the polarization vector by exciting a depolarizing resonance with a field configuration which is sinusoidally distributed along the orbit². By this, the phase of the exciting field distribution will define the direction of the invariant spin axis, thus, the direction of the polarization vector. For example, each maximum of the exciting fields corresponds to a radial orientation of the final polarization vector (see fig. 1).

NUMERICAL ANALYSIS WITH SYNCHROTRON MOTION

In particular for high energy electrons, the synchrotron motion has to be considered thoroughly for two reasons:

On the one hand, the emission of synchrotron light results in an energy oscillation, such that sideband resonances occur adjacent to the primary integer spin tune resonance.

On the other hand, the synchrotron motion leads to an energy spread of the particles. This energy spread is re-

¹For the spin motion, this is a good assumption, since all contributions of the single particle motion apart from those resulting from the common motion (center of charge) average out to zero.

²More precisely, the field configuration is sinusoidally distributed against the spin phase advance and not along the orbit, as the main dipoles are usually not installed at equidistant positions.

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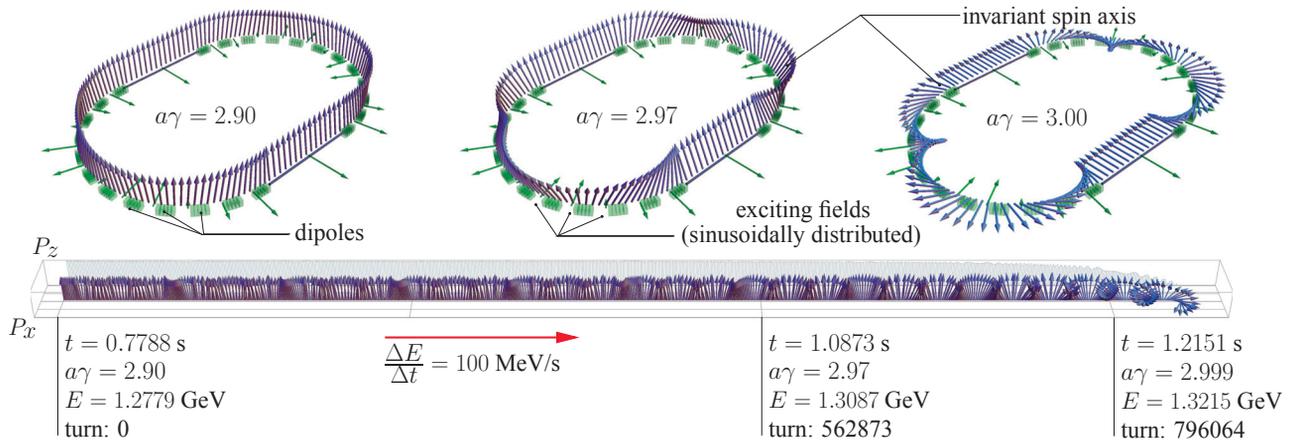


Figure 1: Invariant spin axis in the frame of schematically illustrated dipoles for three particle energies close to and in case of third integer resonance. Below, the polarization vector is shown for an nearly adiabatically energy increase towards the resonance, while being excited by an oscillating horizontal field with a frequency which is a harmonic of the revolution frequency.

sponsible for the effect of depolarization since it results in stochastically distributed single particle spin tunes. Accordingly, the motion of the particle spins becomes incoherent and the length of the polarization vector becomes smaller and smaller — note that the asymptotic degree of polarization is zero in case of a resonance.

The numerical analysis consists of a Runge-Kutta integration of the Thomas-BMT equation with an adaptive control of the step size. In order to enhance the performance, all cores of the computer have been used by multithreading. By this, each thread is associated with the calculation of one particle.

Using the mentioned method, the anterior synchrotron sideband shows to have mainly negative consequences for our purpose. The decision was undertaken to excite the resonance only shortly before the main integer resonance occurs. Thus, the influence of the synchrotron sidebands can be minimized. Figure 2 shows the results of numerical evaluations of the vertical polarization for a single particle in case of excitation during the whole time compared to an excitation just close to the resonance. Using an energy increase of $\Delta E/\Delta t = 100$ MeV/s, a significant impact of the synchrotron sideband can be seen at $t \approx 1.015$ s for one particle and additionally causes finally a full loss of the polarization degree (here, the vertical polarization is shown for an average over 100 particles). The excitation is approximated by a sinusoidal horizontal magnetic field $b_x \sim \cos(1 + a\gamma) \cdot \omega_{\text{rev}}t + \phi$, where ω_{rev} represents the revolution frequency. Hence, the direction of the polarization vector in the horizontal plane depends on the phase ϕ .

When the particle's kinetic energy reaches the third integer spin tune resonance, the energy is kept constant using a constant acceleration power inside the cavities and for fixed magnetic fields strengths. Accordingly, the polarization vector follows the exciting field 180° phase shifted. If the excitation is not continued at the final energy, the degree of polarization becomes lost due to the energy spread

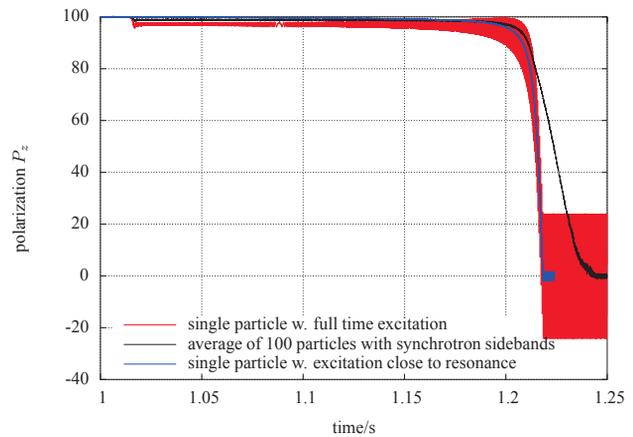


Figure 2: Comparison of the vertical polarization for two particles, first being excited during the whole interval, second being excited close to the resonance at $t \approx 1.22$ s, namely $a\gamma = 3$. Averaging for example for 100 particles results in a vertical polarization of zero and even more in a vanishing degree of polarization (not shown here).

of the particles. Assuming an energy uncertainty in the order of $\Delta E/E = 0.0005$, the degree of polarization of an initially longitudinally oriented ensemble of spins then decays exponentially within approximately $100 \mu\text{s}$ (see fig. 3) according to

$$P_{x,s}(t) = P_0 \cdot \exp\left(-\frac{1}{2} \left(2\pi \frac{\Delta E}{E} \omega_{\text{rev}} \cdot t\right)^2\right) \quad (1)$$

It is remarkable to notice, that in contrast to the previous case, the polarization loss is almost negligible if the integer resonance is intentionally excited. Figure 4 shows the longitudinal degree of polarization for an fulfilled integer spin tune resonance condition. In the latter case, the time scale

of widely preserved polarization is very long compared to the one without excitation.

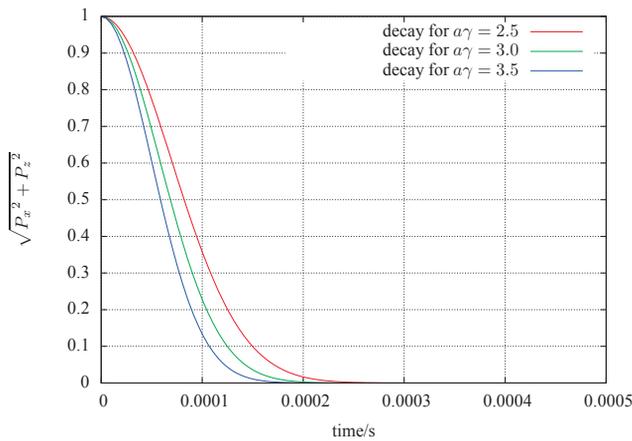


Figure 3: Decay of the longitudinal degree of polarization without exciting the resonance appropriately.

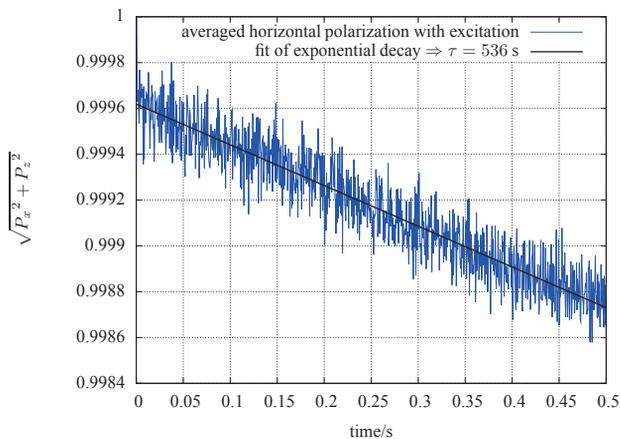


Figure 4: Decay of the longitudinal degree of polarization with an excitation of the integer spine tune resonance.

EMPIRICAL RESULTS AT ELSA AND OUTLOOK

In 2009, we accomplished a demonstration test at the ELSA stretcher ring. We used an energy increase of 100 MeV/s in order to guarantee an adiabatically motion of the polarization vector according to that one of the invariant spin axis. During this process, we applied sinusoidally distributed horizontal fields via 24 installed dipole correctors. Then we extracted the beam towards the external experiment where a Møller-polarimeter enables measurements of the longitudinal polarization. By shifting the phase of the field distribution, we were able to obtain different degrees of polarization in the horizontal plane without using solenoids inside or outside the ring. As the degree of polarization is measured along the direction of the momentum,

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a lower measured degree of polarization has to be interpreted as a rotation of the polarization vector in the horizontal plane. Figure 5 shows the measured longitudinal polarization for different phases of the horizontal field distribution. During the demonstration tests, we were faced

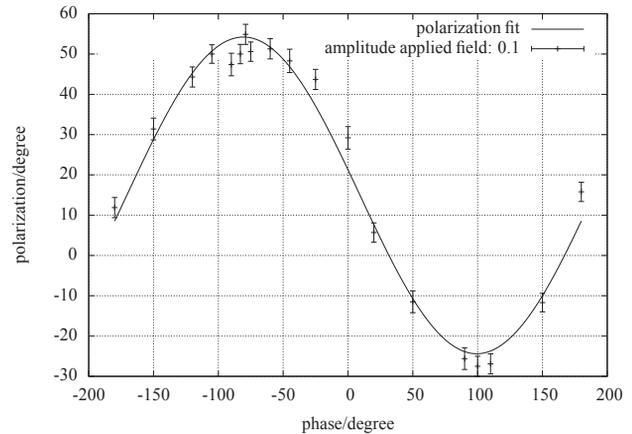


Figure 5: First results of the measured degree of polarization against the phase ϕ of the exciting field distribution.

with a negative effect when using the vertical dipole correctors for applying sinusoidally distributed fields. A vertical displacement of the beam inside each quadrupole appeared due to the vertical kick of the correctors. According to this vertical displacement, an additional horizontal magnetic field act on the beam and, in turn, on the spin motion. Including all corrector and quadrupole fields, this lead to an entire horizontal field distribution which is not sinusoidally anymore. In [2] we presented an algorithm to solve this problem. Having modified the orbit response matrix for this purpose, we were able to calculate the specific kick angles of the dipole correctors in order to gain the desired field distribution. It has been shown that the required kick strength results in the need of more powerful corrector supplies, which have been installed recently. This enables more detailed studies regarding horizontal polarization in the stretcher ring at higher spin tunes of $a\gamma = 4, 5, 6, 7$. Moreover, the presented method can also be carried out using rf-dipoles for which the horizontal field oscillates in phase with the spin precession, in other words, with an integer multiple of the revolution frequency. Hereby, the phase determines the phase of the spin precession and thus, the direction of the polarization vector at all positions.

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