INTRODUCTION

The injection energy upgrade from 181 MeV to 400 MeV is scheduled in 2013 in J-PARC Rapid Cycling Synchrotron (RCS) [1] to decrease the space charge effect. Longitudinal beam emittance at the extraction should be optimized to avoid the beam loss after MR injection. In order to match the longitudinal emittance shape between the RCS and the MR, it is desirable to enlarge the longitudinal emittance in the RCS. We have performed the particle tracking simulation for the controlled longitudinal emittance blow-up in the RCS.

EMITTANCE BLOW-UP BY PHASE MODULATION

For the emittance blow-up by the phase modulation, the high frequency cavity voltage is added to the fundamental acceleration voltage. The total rf voltage $V_t$ is written as

$$ V_t = V_0 \sin h_0 \omega_{\text{rev}} t + V_h \sin (h_b \omega_{\text{rev}} t + \psi(t) + \psi_b), \quad (1) $$

where $V_0$ and $h_0$ are amplitude and harmonic number of the fundamental acceleration voltage, and $\omega_{\text{rev}}$ is the angular revolution frequency of a synchronous particle. $V_h$ and $h_b$ are amplitude and harmonic number of the high frequency cavity. $\psi_b$ is the phase offset value. The phase of the high frequency cavity is modulated by

$$ \psi(t) = \Delta \phi_{\text{mod}} \sin \omega_{\text{mod}} t, \quad (2) $$

where $\Delta \phi_{\text{mod}}$ is the “amplitude” of the phase modulation, and $\omega_{\text{mod}}$ is the angular modulation frequency.

The blow-up characteristics are categorized by the ratio of the modulation frequency $\omega_{\text{mod}}$ to the synchrotron frequency $\omega_s$. It is called 'resonant regime [3]' in the case of $\omega_{\text{mod}}/\omega_s < 5$, and is called 'noise regime [4, 5]' in the case of $\omega_{\text{mod}}/\omega_s > 10$. Although we have performed the particle tracking simulations for both regimes, we describe the results only for the resonant regime because we could not find appropriate blow-up condition for the noise regime.

In the simulation, the beam loading effects and the space charge effects are not included because we want to see the modulation effect clearly. And, the amplitude of the high frequency cavity voltage is defined to keep the bucket height constant with respect to the momentum deviation.

Figure 1 shows the simulation result of the phase space and the bunch shape at the RCS extraction without any modulation. The thin circle line drawn in the phase space indicates the beam emittance of 5 eVs, this is the design value for the RCS rf bucket.

![Figure 1: The longitudinal beam emittance and the bunch shape at the RCS extraction. No blow-up is applied.](image)

From the parameter search on the resonant regime, we can find the conditions of $\omega_{\text{mod}}/\omega_s = 4$ and $h_b/h_0 = 20 \sim 30$ are suitable for the controlled emittance blow-up in the RCS. The simulation results are shown in Figs. 2-7, and the simulation conditions are listed in Table 1. The bucket height of the high frequency cavity is expressed in terms of the momentum difference with respect to the synchronous
The bucket height is set as the beam emittance is enlarged around 10 eVs at the RCS extraction.

Table 1: Simulation Conditions for Phase Modulation in the Resonant Regime

<table>
<thead>
<tr>
<th>Start</th>
<th>( h_b/h_0 )</th>
<th>Bucket height</th>
<th>( \Delta \phi_{\text{mod}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 2</td>
<td>16 ms</td>
<td>20</td>
<td>0.19 %, 180 deg.</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>16 ms</td>
<td>30</td>
<td>0.21 %, 180 deg.</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>18 ms</td>
<td>20</td>
<td>0.23 %, 180 deg.</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>18 ms</td>
<td>30</td>
<td>0.25 %, 180 deg.</td>
</tr>
</tbody>
</table>

Figures 2-5 show the beam emittance and the bunch shape at the RCS extraction. The thin circle line drawn in the phase space indicates the beam emittance of 10 eVs, this is the suitable value for the MR rf bucket. Figures 6 and 7 show the variation of the beam emittance and the high frequency cavity voltage in the latter part of the RCS acceleration period, the solid line shows the case of \( h_b/h_0 = 20 \) and the dotted line shows the case of \( h_b/h_0 = 30 \). The phase modulation is added from 16 ms (Figs. 2, 3 and 6) or 18 ms (Figs. 4, 5 and 7) in 20 ms acceleration period.

The beam emittance in the case of \( h_b/h_0 = 30 \) is dilute smoother than in the case of \( h_b/h_0 = 20 \). However, 70 ~ 80 % higher modulation voltage \( V_i \) is needed to achieve the same beam emittance in the case of \( h_b/h_0 = 30 \) as shown in Figs. 6 and 7. The starting time of the phase modulation does not make a big difference for the bunch shape, but 40 % higher modulation voltage is needed in the case of 18 ms starting time compared with the 16 ms case.

Figure 6: The variation of the longitudinal beam emittance and the high frequency voltage (16 ms start).

Figure 7: The variation of the longitudinal beam emittance and the high frequency voltage (18 ms start).

Summarizing the results, the condition around \( h_b/h_0 = 30 \) is suitable for the RCS controlled emittance blow-up using the phase modulation.

**EMITTANCE BLOW-UP BY RESONANCE OVERLAP**

In the case of the resonance overlap method, two and more terms of the high frequency voltage are added to the fundamental one. The total rf voltage \( V_i \) is written as

\[
V_i = V_0 \sin \omega_{\text{revs}} t + \sum_{N=1}^{N_e} V_{bN} \sin(\omega_{bN} t + \psi_{bN}) ,
\]

where \( N \) is a serial number corresponding to each high frequency voltage. \( \omega_{bN} \) is the angular frequency of each high frequency voltage and \( \psi_{bN} \) is the phase offset. \( \omega_{bN} \) has the frequency offset \( \Delta \omega_{bN} \) from the revolution frequency of the synchronous particle as

\[
\omega_{bN} = \omega_{\text{revs}} \Delta \omega_{bN} .
\]

The resonance overlap occurs when the following condition between \( V_{bN} \) and \( \Delta \omega_{bN} \) is satisfied [7] as

\[
eV_{bN} \geq \frac{\pi \hbar \omega_{\text{revs}}^2 \omega_{bN}^2}{2 \eta_s} \sigma_{bN}^2 \frac{\Delta \rho_{bN}}{\rho_s} ,
\]

where \( e \) is the elementary electric charge, \( \beta_s \) and \( E_s \) are the ratio of the velocity to the speed of light and the total energy of the synchronous particle, and \( \eta_s \) is the slippage factor. This overlap condition is not so strict, the emittance blow-up occurs on a little bit smaller value of \( V_{bN} \) in Eq. (5).

In the simulation, \( \Delta \omega_{bN} \) is set that each high frequency rf bucket is separated with constant momentum deviation \( \Delta \rho_{bN} \) from the synchronous particle as

\[
\Delta \omega_{bN} = \hbar \omega_{\text{revs}} \eta_s \left( \frac{\Delta \rho_{bN}}{\rho_s} \right) .
\]

From the parameter search on the resonance overlap, we can also find \( h_b/h_0 = 20 \sim 30 \) is suitable for the RCS. The simulation results are shown in Figs. 8-13, and the simulation conditions are listed in Table 2. We use three high
frequency voltages, that is $N_c = 3$. The momentum deviation of each rf bucket is set as $\Delta p_{s1,2,3}/p_s = 0, \pm 0.3 \%$. The bucket heights of each high frequency voltage are the same, and they are set as the beam emittance is enlarged around 10 eVs at the RCS extraction.

The expressions of the simulation results in Figs. 8-13 are same as mentioned at the phase modulation case. From the simulation results, the resonance overlap also realizes the controlled beam emittance blow-up in the RCS. However, there are some different characteristics from the phase modulation.

The bunch shape in the case of $h_b/h_0 = 20$ is wider than in the case of $h_b/h_0 = 30$, this is the opposite characteristics compared with the phase modulation. Furthermore, almost same voltage is necessary to achieve 10 eVs emittance between the $h_b/h_0 = 20$ and $h_b/h_0 = 30$ as shown in Figs. 12 and 13, the starting time of the resonance overlap does not make a big difference on the voltage compared with the phase modulation case. It seems that the emittance blow-up is sensitive to a small difference on the condition of the resonance overlap.

Summarizing the results, the condition of $h_b/h_0 = 20$ is suitable for the RCS controlled emittance blow-up using the resonance overlap.

In comparison with the phase modulation, the resonance overlap needs more high frequency voltage (note that three high frequency voltages are used in the resonance overlap), but the frequency is lower than the case of the phase modulation. In order to choose which is better method, the construction feasibility study of the high frequency rf system is needed. Furthermore, the simulation with the space charge effects and the beam loading effects should be done to evaluate the precise matching between the RCS and the MR.

**SUMMARY**

We have investigated the controlled longitudinal beam emittance blow-up by the phase modulation and the resonance overlap in the J-PARC RCS, and it is found that the emittance blow-up is possible for both methods. The feasibility study for the high frequency cavity is needed to realize the emittance blow-up in the RCS.

**REFERENCES**