MODELING OF QUASI-PHASE MATCHING IN AN APERIODIC CORRUGATED PLASMA WAVEGUIDE FOR HIGH-EFFICIENCY DIRECT LASER ELECTRON ACCELERATION *

M.-W. Lin and I. Jovanovic †, Department of Mechanical and Nuclear Engineering, The Pennsylvania State University, University Park, PA 16802, U.S.A.

Abstract

Direct laser acceleration (DLA) of charged particles using the axial electric field of a radially polarized intense laser pulse is promising for realization of compact accelerators required in security and medical applications. The implementation of guided propagation of laser pulses over long distances and the phase matching between electrons and laser pulses may limit the performance of DLA in reality. A corrugated plasma waveguide could be applied to extend the laser beam propagation distance and for quasi-phase matching between laser and electron pulses for net acceleration. To accelerate electrons from a low initial energy (for example, 5 MeV from a photoinjector gun) up to hundreds of MeV, an aperiodic corrugated plasma waveguide with successive increase of on-axis density modulation period is needed. We conducted particle-in-cell simulations to design the appropriate aperiodic plasma structure for DLA. For each section of the corrugated waveguide, the dependence of density modulation period on the initial electron energy and laser pulse intensity is investigated. The simulation results are guiding the design of proof-of-principle experiments for compact, tabletop DLA.

INTRODUCTION

Direct laser acceleration (DLA) of electrons, such as inverse Cherenkov acceleration [1, 2, 3], has the potential to meet the requirements for future compact accelerator-driven systems. In DLA, electrons are accelerated by the axial component of the electric field of a focused, radially polarized laser pulse. The axial field amplitude scales as the square root of the laser peak power [2, 3], and is estimated to be 77 GV/m for 800 nm laser wavelength with peak power of 0.5 TW and 8.5 µm mode radius. Therefore, field gradients on the order of hundreds of MeV/cm are expected even below TW peak power from the drive laser.

Optical guiding in DLA using a preformed plasma waveguide [2] can extend the accelerating distance. To compensate for the phase mismatch effect in DLA, corrugated waveguide techniques [3, 4] with axially periodic plasma density modulation could be used to quasi-phase match (QPM) the laser and electron pulses. The QPM period depends on the electron velocity, and changes rapidly for electron beams with low initial kinetic energies (<20 MeV). Therefore, an aperiodic corrugated waveguide structure is required to achieve effective QPM DLA with low initial electron energies.

In this work we studied the optimal plasma structures for DLA in corrugated plasma waveguides with a range of initial electron energies. A test electron particle model has been developed to search for optimal QPM conditions. Next, particle-in-cell (PIC) simulations [5] have been performed and compared with the results obtained from the test particle method.

ELECTRON TEST PARTICLE MODEL FOR DLA IN CORRUGATED PLASMA WAVEGUIDES

Modelling of DLA is based on the dispersion relation for a guided radially polarized laser pulse in a preformed plasma waveguide [2]. Under paraxial approximation, the refractive index of a guided laser pulse is:

\[ \eta = \left(1 - \omega_p^2/\omega^2 - 8c^2/\omega^2\omega_0^2\right)^{1/2}, \]

where \( \omega_p \) is the plasma frequency, \( \omega \) is the laser frequency, \( \omega_0 \) is the guided mode radius, and \( c \) is the speed of light. The phase velocity \( v_{ph} = c/\eta \) and group velocity \( v_g = c\eta \) characterize the propagation of the guided laser pulse. With plasma \( \eta < 1 \), the phase velocity \( v_{ph} > c \), resulting in the phase mismatch effect between the laser pulse and the electrons. Electrons will fall out of phase by \( \pi \) with respect to the co-propagating laser field after a dephasing length \( L_c = \pi/k_l - k_c \) determined by the laser wave vector \( k_l = 2\pi\eta/\lambda \) and \( k_c = \omega/v_c \) as the equivalent wave vector for the electron beam with velocity \( v_c \). Figure 1 shows the dependence of dephasing length \( L_c \) on electron kinetic energy for three plasma electron densities \( n_e \) with \( \omega_0 = 8.5 \) μm and \( \lambda = 800 \) nm.

![Figure 1](image-url)
The test particle model is developed by solving coupled relativistic equations of motion of a single electron with given initial parameters. The electron trajectory and energy $(\gamma m_e c^2)$ are calculated from $d\mathbf{p}_e/dt = q_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B})$ and $m_e c^2 d\gamma/dt = q_e (\mathbf{v}_e \cdot \mathbf{E})$ with the Lorentz factor $\gamma$, the electron momentum $\mathbf{p}_e = \gamma m_e \mathbf{v}_e$, and electron’s charge $q_e$ and rest mass $m_e$. The electromagnetic field of a co-propagating, guided, radially polarized laser pulse in a corrugated plasma waveguide is considered as the driving field [2]. For a guided radially polarized laser pulse, the electric field $\mathbf{E} = \hat{r}E_r + \hat{z}E_z$, in which the radial field $E_r$ is:

$$E_r = E_0 \theta (r/w_0) \exp[-(r/w_0)^2] \times \exp[-2\ln 2 (z-z_p)^2/L_p^2] \cos(\phi),$$

and the axial field $E_z$ is:

$$E_z = E_0 \theta^2 (1 - r^2/w_0^2) \exp[-(r/w_0)^2] \times \exp[-2\ln 2 (z-z_p)^2/L_p^2] \sin(\phi),$$

for a Gaussian pulse at position $z_p$, with the phase $\phi$, the beam diffraction angle $\theta = \lambda/\pi w_0$, and the pulse length $L_p$. The characteristic field amplitude $E_0$ relates to laser peak power $P_w$ as $E_0 = (8c\mu_0 P_w/\pi w_0^2 \theta^2)^{1/2}$. The azimuthal magnetic field is defined as $B_\theta = (\eta_0/c) E_r$. The phase of the field is represented by $\phi = \omega (t - z_0/c)$. The change of phase and position of the laser pulse can be described by $d\phi/dt$ and $dz_p/dt = v_p(z_p)$. To simulate a corrugated plasma waveguide, the axial plasma electron density is chosen as a function of axial position $z$ by:

$$n_{c0}(z) = n_{c,L} + (n_{c,H} - n_{c,L}) \times H(z - \lambda_{L,n}) [1 - H(z - \lambda_{m,n})],$$

where $n_{c,L}$ and $n_{c,H}$ are the plasma electron densities in low- and high-density regions, and $\lambda_{m,n} = \lambda_{L,n} + \lambda_{H,n}$ is the length of $n$th-modulation period composed of low- and high-density regions of lengths $\lambda_{L,n}$ and $\lambda_{H,n}$, respectively. $H$ is the Heaviside function used to define a sharp axial density change. This axial density distribution function $n_{c0}(z)$ is used to update the axial refraction index $\eta(z)$, on-axis plasma frequency $\omega_{pe}(z)$, and the laser pulse group velocity $v_p(z) = c \eta(z)$. Initial parameters include the electron position $z_0=0$ and $\theta_0=0$, radial momentum $P_{r0}=0$, pulse position $z_{p0}=2.4 \mu m$, and field phase $\phi_0=3 \pi$. 2 mm plasma waveguides with default plasma electron densities $n_{e,L}=2.5 \times 10^{18} \text{ cm}^{-3}$ and $n_{e,H}=1.25 \times 10^{19} \text{ cm}^{-3}$ are considered. The initial axial momentum $P_{z0}$ and $\gamma_0$ factor are assigned according to the desired initial kinetic energy $T_0$. A 0.5 TW laser pulse with $L_p=6 \mu m$ (corresponding to a pulse duration of 20 fs) and $E_{z,max} = E_0\theta^2 \approx 77 \text{ GV/m}$ is considered with the dispersion relation determined by $w_0=8.5 \mu m$ and $\lambda=800 \text{ nm}$.

The same laser wavelength and waveguide diameter are used in all simulations described. Figure 2(a) shows the dependence of electron energy gain $\Delta T$ on axial position $z$ for DLA with initial kinetic energies $T_0=5-100 \text{ MeV}$ with constant density modulation periods in waveguides. The modulation periods are determined by the dephasing length $L_c$ (Fig. 1) for chosen $T_0$ energies. The results indicate that with constant density modulation one can only achieve QPM DLA for electrons with higher initial kinetic energies ($T_0=40 \text{ MeV}$ and $100 \text{ MeV}$ in this example). In contrast, aperiodic density modulation is required for DLA of electrons with lower initial energies ($T_0=5 \text{ MeV}$ and $15 \text{ MeV}$ in this example), as shown in Fig. 2(b). It is particularly important to accurately set the plasma structure period in the first few sections of the waveguide so that the electrons can remain in the acceleration phase. The results reflect the rapid change of dephasing length in the range of $T_0 \leq 20 \text{ MeV}$. As a result, the modulation period should be increased according to the increased electron energy and then gradually converge to the dephasing length $L_c$ at higher electron energies. We find it sufficiently accurate to approximate each section length $L_{d,n}$ by the average of its initial dephasing length $L_{c,i}$, obtained for the initial electron kinetic energy $T_{0,i}$ of that section, and the final dephasing length $L_{c,f}$, obtained by the approximated final kinetic energy $T_{0,f}$ after acceleration by $L_{c,i}$. Thus $L_{d,i} = 1/2 (L_{c,i} + L_{c,f})$ provides a good approximation for designing an efficient aperiodic DLA structure.

With optimally designed plasma structures, the axial acceleration phase region for QPM DLA can be investigated by injecting test electrons with various initial axial positions $z_0$ and phases $\phi_0$ along the central axis. Figure 3

---

**Figure 2**: Dependence of electron energy gain $\Delta T$ on propagation distance $z$ for electrons with $T_0=5, 15, 40$ and $100 \text{ MeV}$ in plasma waveguides with (a) constant and (b) aperiodic density modulation (only applied for $T_0=5$ and $15 \text{ MeV}$). The other parameters are given in text.
shows the final electron energy gain $\Delta T$ as a function of initial phase $\phi_0$ for various initial electron kinetic energies $T_0$. The results indicate that, in general, DLA with higher initial electron energies will exhibit a broader acceleration phase region. In the case of $T_0=5$ MeV, the acceleration phase region width is about 0.4$\pi$ around the optimal phase $\phi_0 = 3\pi$ and it increases to $\pi$ for $T_0=100$ MeV. Particles with $\Delta T$ around 0 in Fig. 3 exhibit rapid energy oscillations throughout the propagation but cannot accumulate energy gain. The acceleration region shifts $\pi/2$ from the initial acceleration region. For example, electrons with initial phase $\phi_0=2.5\pi-3\pi$ decelerate initially, but enter acceleration phases later in the process. The process is reversed when $\phi_0=3.5\pi-4\pi$.

**PIC SIMULATION OF DLA**

3D particle-in-cell (PIC) simulations [5] have also been performed. Each simulation is performed in a moving frame co-propagating with the laser pulse. The size of the simulation box is 20.27 $\mu$m in the axial direction $z$ and 43.2 $\mu$m $\times$ 43.2 $\mu$m in the transverse direction $r$. Interlacing waveguide sections and neutral hydrogen gas sections are defined. The waveguide sections have a transverse plasma density profile in which density increases quadratically along $r$ from the central density of $n_{e,0}=n_{e,L}=2.5\times10^{18}$ cm$^{-3}$. The density profile defines a laser-guided mode radius of $w_0=8.5$ $\mu$m. The neutral hydrogen gas sections have a uniform density distribution with density $n_0=n_{e,H}=1.25\times10^{19}$ cm$^{-3}$, and are ionized with the passage of a 20 fs, 0.5 TW, radially polarized laser pulse. In the tests of dephasing length as shown in Fig. 4(a) for DLA in non-corrugated waveguides (only a single waveguide section is defined in space), we find the factor 8 of the waveguide dispersion term in the refractive index $\eta$ of Eq. (1) should be modified to approximately 5. With this correction, the results from our test particle model agree well with the results from 3D PIC simulations. Figure 4(a) and 4(b) show QPM DLA for $T_0=5$ MeV, 15 MeV, and 40 MeV in plasma waveguides with proper plasma structure for QPM DLA. The results also show that aperiodic corrugations in plasma waveguides are required for DLA with lower electron initial energies, while a constant density modulation period is applicable for the case of higher electron initial energies.

**CONCLUSION**

Aperiodic modulation of corrugated plasma waveguides are required for DLA with low electron kinetic energies ($\lesssim 20$ MeV). For such low injected electron energies, DLA dephasing length changes rapidly with the increased electron energy. Therefore, the plasma structure should be varied accordingly so that electrons can remain in the acceleration phase throughout the propagation distance. In addition, DLA with low initial electron energies will also result in small axial acceleration regions after the QPM process. Our test particle model provide a straightforward method to estimate QPM conditions for DLA, as confirmed by 3D PIC simulations.

**REFERENCES**